

**Finance and Economics Discussion Series  
Divisions of Research & Statistics and Monetary Affairs  
Federal Reserve Board, Washington, D.C.**

**Cheapest-to-Deliver Pricing, Optimal MBS Securitization, and  
Market Quality**

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**2021-031**

Please cite this paper as:

Huh, Yesol, and You Suk Kim (2021). “Cheapest-to-Deliver Pricing, Optimal MBS Securitization, and Market Quality,” Finance and Economics Discussion Series 2021-031. Washington: Board of Governors of the Federal Reserve System, <https://doi.org/10.17016/FEDS.2021.031>.

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# Cheapest-to-Deliver Pricing, Optimal MBS Securitization, and Market Quality

Yesol Huh and You Suk Kim\*

First draft: June 16, 2020

Last updated: April 19, 2021

## Abstract

We study optimal securitization and its impact on market quality when the secondary market structure leads to cheapest-to-deliver pricing in the context of agency mortgage-backed securities (MBS). A majority of MBS are traded in the to-be-announced (TBA) market, which concentrates trading of heterogeneous MBS into a few liquid TBA contracts but induces adverse selection. We find that lenders segregate loans of like values into separate pools and tend to trade low-value MBS in the TBA market and high-value MBS outside the TBA market. We then present a model of optimal securitization for agency MBS. Lenders do not internalize the negative impact of such pooling and trading strategies on TBA market quality and thus create too many high-value MBS, which leads to more heterogeneity in MBS values, more adverse selection, and lower TBA liquidity. Lastly, we provide empirical evidence consistent with model predictions on how MBS pooling changes with trading costs and underlying loan value dispersion and how pooling practices affect MBS heterogeneity and TBA market adverse selection.

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# 1 Introduction

The literature on security design has explained various features of asset-backed securities as the optimal response of issuers to mitigate asymmetric information about underlying assets. For example, cash flows are pooled and tranching to create low-risk and liquid securities (DeMarzo, 2005). Also, higher underlying asset qualities are signalled by creating larger equity tranches (Begley and Purnanandam, 2017; Flynn, Ghent, and Tchisti, 2020) and holding loans longer before securitization (Adelino, Gerardi, and Hartman-Glaser, 2019). By mitigating information frictions, such security designs facilitate trading and enhance the liquidity of the security. In fact, such designs are adopted by a wide range of securities such as commercial mortgage-backed securities (MBS), non-agency MBS, and securities backed by auto loans and credit card loans.

However, these security design theories do not explain the design of agency MBS, which account for a vast majority of securitized assets since the Global Financial Crisis (GFC).<sup>1</sup> Since the credit risk of agency MBS is guaranteed, asymmetric information about underlying asset values is less important compared with other asset-backed securities. Further, a majority of agency MBS trade as pass-through securities, their cash flows are not tranching, and issuers do not have a signalling mechanism.<sup>2</sup> In this paper, we show that the unique trading structure of the agency MBS market heavily influences an important aspect of agency MBS design. Moreover, in contrast to the aforementioned security design literature, we find that the optimal agency MBS design chosen by lenders negatively affects overall market quality. Understanding agency MBS design and its impact on market quality is important because of its broad implications beyond the agency MBS market. For example, Huh and Kim (2020) find that agency MBS liquidity affects mortgage rates and monetary policy transmission.

An important aspect in the market for agency MBS is the to-be-announced (TBA) forward contract, which accounts for more than 90% of agency MBS trading. In a TBA contract, parties agree on limited MBS characteristics instead of specifying the CUSIP. This trading structure is credited with generating liquidity by concentrating trading of MBS with heterogeneous values into a handful of thickly traded TBA contracts. Sellers can also trade MBS in a much less liquid specified-pool (SP) market by specifying the individual CUSIP. Thus, sellers will sell only the cheapest securities through the TBA market, leading to the cheapest-to-deliver pricing and adverse selection in this market. Large heterogeneity in MBS value will

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<sup>1</sup>According to SIFMA, in 2019, there were \$1.6 trillion asset-backed securities outstanding and \$10.2 trillion MBS outstanding. Agency MBS accounts for \$8.8 trillion. <https://www.sifma.org/resources/research/fixed-income-chart/>

<sup>2</sup>Real estate mortgage investment conduits (REMICs) pool pass-through securities and tranche them to create multi-class securities. Fannie Mae and Freddie Mac issue REMICs by pooling and tranching agency pass-through securities. REMIC issuance amounts vary over time, but during our sample period, 10–20% of agency MBS pass-through securities end up as collateral in REMICs. Downing et al. (2009) shows that pass-through MBS that are used as collateral in REMICs tend to be of lower quality due to similar cheapest-to-deliver incentives.

exacerbate adverse selection and limit the liquidity benefit of the TBA market structure.

This paper studies theoretically and empirically how the agency MBS market trading structure affects securitization, MBS heterogeneity, and, ultimately, TBA market adverse selection and liquidity. We show that because of the cheapest-to-deliver incentives of the TBA market, lenders pool high-value loans by themselves to create high-value MBS and sell those MBS in the SP market. Lenders do not take into account the negative impact of such pooling and trading strategies on TBA market quality; thus, they create too many high-value MBS, which make MBS values more heterogeneous, increase adverse selection in the TBA market, and make the TBA market less liquid. We also show theoretically and empirically how pooling and adverse selection change with trading costs and loan value dispersion.

We begin our analysis by documenting that pooling is far from random and is strongly affected by the cheapest-to-deliver incentive in the TBA market. Lenders tend to create high-value MBS backed only by high-value loans and low-value MBS backed by all other loans. High-value MBS account for at least 20% of MBS issuance, and such a pooling strategy endogenously increases MBS value heterogeneity. We then find that high-value MBS tend to be traded via SP to receive a price higher than the cheapest-to-deliver TBA price despite higher SP trading costs, whereas low-value MBS tend to be traded via TBA. These findings suggest that lenders create high-value MBS to avoid the cheapest-to-deliver pricing in TBA, but such pooling leads to partial unravelling of the TBA market.

We then model lenders' pooling and trading problems to understand how their pooling and trading strategies determine the TBA price, liquidity, and adverse selection in equilibrium. The model has two stages. In the first stage, each lender is endowed with a distribution of mortgages and pools the mortgages into securities. Specifically, each lender chooses a "degree of pooling," the share of loans to pool together into a single MBS, which determines the cutoff loan value. Loans with values up to the cutoff value are pooled into the single MBS ("cheapest-to-deliver MBS"), and loans with values greater than the cutoff are sold individually as MBS backed by a single loan ("high-value MBS"). Thus, a higher degree of pooling leads to lower MBS value heterogeneity. Each lender pays an issuance cost, which is lower for a higher degree of pooling because it would lead to fewer individual MBS.

In the second stage, lenders sell MBS created in the first stage in either the TBA or the SP market. The TBA market price does not depend on the fundamental value of the specific MBS but is the expected value of the MBS traded in the TBA market. Lenders receive the fundamental value but pay higher trading costs in the SP market. The SP/TBA trading cost difference increases in the TBA trading volume to allow liquidity to depend on the trading volume of each market. Lenders always trade the cheapest-to-deliver MBS

in the TBA market but trade high-value MBS in the SP market as long as the realization of the stochastic component of the SP trading cost is not too high. This sorting of trades decreases TBA price, lowering revenues for all lenders. However, because each lender takes TBA price and volume as given, lenders do not internalize this negative externality, leading to partial unravelling of the TBA market.

Lenders choose the degree of pooling in the first stage, which determines MBS value heterogeneity and the degree of adverse selection in the second stage. A lower degree of pooling gives more flexibility for the lender to choose between the TBA and SP markets for the marginal loan but increases the issuance cost. A social planner that maximizes lenders' revenues would pool all mortgages into a single MBS so that all MBS are homogeneous and trade only through the TBA market. The social planner minimizes SP trading costs because they are a dead-weight loss. In contrast, lenders choose a lower degree of pooling in equilibrium, which leads to higher MBS value heterogeneity, more SP trading volume, and greater total trading costs paid by lenders. Because an individual lender alone does not affect the equilibrium, a lender's pooling decision does not internalize the social cost from creating high-value MBS and trading them in the SP market.

To understand the impacts of potential policy changes, we study how the equilibrium changes with model parameters. We show that when the SP/TBA trading cost difference increases or when loan value dispersion decreases, the equilibrium degree of pooling and TBA liquidity increase, and adverse selection decreases. The intuition is as follows. Both an increase in the SP/TBA trading cost difference and a decrease in loan value dispersion would increase the relative payoff from trading in the TBA market for MBS with sufficiently high value. As more high-value MBS are traded in the TBA market, adverse selection will decrease, and TBA market liquidity will increase. Lenders then will choose a higher degree of pooling, increasing the equilibrium degree of pooling, which further reduces adverse selection and increases liquidity in the TBA market.

We then empirically test two of our comparative statics results. Using three different setups, we first test the model predictions on how pooling changes with loan value dispersion and with the trading cost difference between the SP and TBA markets. In the first setup, we compare MBS with different coupons as loan value dispersion is higher for MBS with higher coupons because of their higher refinancing incentives. For the second setup, we use the fact that loan value dispersion decreased suddenly during the 2013 taper tantrum episode because a sharp increase in the mortgage rate decreased refinancing incentives. The last setup uses the fact that Fannie Mae TBA markets are much more liquid than Freddie Mac TBA markets, making the SP/TBA trading cost differential higher for Fannie Mae. We show evidence that empirical pooling patterns are consistent with the model predictions in all three setups.

Lastly, we test using two different setups the model prediction that a lower degree of pooling leads to

more severe adverse selection in the TBA market. We measure the degree of adverse selection as a relative difference in value between MBS traded in the TBA and SP markets. Our first setup compares the degree of adverse selection for the Fannie Mae and Freddie Mac TBA markets with the Ginnie Mae II multi-issuer TBA market. Under the Ginnie Mae II program, Ginnie Mae issues only one MBS per coupon per month, and all lenders contribute their loans to the single MBS, effectively pooling all loans together. We show that adverse selection is far lower for Ginnie Mae TBA, suggesting that a higher degree of pooling leads to less adverse selection. In our second setup, we compare degrees of adverse selection across MBS with different coupons. Because of higher loan value dispersion, MBS with higher coupons should have lower degrees of pooling, leading to more adverse selection. We confirm this is true in the data.

Our findings suggest that the TBA market structure heavily influences lenders' pooling, which determines MBS heterogeneity and affects agency MBS market quality. This is not to say that the TBA market structure is undesirable or makes the MBS market illiquid. Instead, our results highlight that lenders' optimal pooling could make the TBA market illiquid in certain market environments, which suggests that policymakers should consider how future policy changes will affect pooling incentives. For example, ever since the GSEs went into conservatorship during the GFC, there have been discussions to reform the housing finance system. Many housing finance reform proposals still wish to keep the TBA market liquid due to its large liquidity benefits. Our results show that loan value heterogeneity should be limited to preserve liquidity in the TBA market, for example, by providing an explicit government guarantee on credit risk.

This paper is related to a growing number of papers that study MBS market structure, TBA liquidity, and their effects. Vickery and Wright (2013) review institutional details of the TBA market. Bessembinder, Maxwell, and Venkataraman (2013) show that TBA trading costs are much lower (1 bp) than SP trading costs (40 bps). Gao, Schultz, and Song (2017) study impacts of the TBA market on SP market liquidity. Schultz and Song (2019) find that post-trade transparency in the TBA market decreased TBA trading costs for institutional investors. Huh and Kim (2020) find that a liquid TBA market lowers mortgage rates. We add to this literature by studying how the MBS market structure affects lenders' pooling and its impacts on market quality.

Three contemporaneous papers in this literature are closely related to our paper. First, Li and Song (2020) theoretically study how TBA-like market structure can improve liquidity by concentrating trading of heterogeneous securities. Second, Fusari, Li, Liu, and Song (2020) study asset pricing implications of TBA market structure, examining how MBS heterogeneity affects MBS pricing and trading. However, these two papers are silent about sources of MBS heterogeneity. We complement this paper by examining how lenders'

pooling determines MBS heterogeneity and how important policies like housing finance reforms would affect MBS heterogeneity and market quality. Moreover, we test model predictions empirically, whereas Li and Song (2020) only provide theoretical results.

Third, An, Li, and Song (2020) study lenders' pooling decisions theoretically and test model predictions on pooling decisions. While they consider a more general pooling strategy, their focus is mostly on pooling decisions themselves. We complement their paper by studying not only the pooling decisions but also how the pooling decisions affects market quality such as adverse selection and liquidity. Our model also allows for trading costs to depend on trading volumes, which allows for feedback between the degree of pooling and liquidity. These additional elements in our paper allow us to understand the contrast between lenders' optimal decisions and the social planner's decision. In fact, An et al. (2020) find lenders receive quantitatively significant benefits from optimal pooling in a partial equilibrium structural model in which the TBA price and volume remain the same. However, our model suggests that in equilibrium, optimal pooling will make lenders worse off as a whole due to its negative externality. This normative analysis is important in assessing the welfare impacts of future policy changes. Moreover, our empirical analysis investigates not only whether pooling practices in the data are consistent with model predictions but also how pooling practices affect adverse selection in the TBA market.

This paper is also related to other works that study designs of asset-backed securities. Asymmetric information between security issuers (or informed intermediaries) and buyers is the main driving force for a majority of these papers.<sup>3</sup> Thus, these papers often involve tranching (DeMarzo, 2005; Friewald, Hennessy, and Jankowitsch, 2016), a signalling component (Adelino et al., 2019), or an equity retention component (Begley and Purnanandam, 2017; Flynn et al., 2020). Also, Glaeser and Kallal (1997) show that pooling and limited disclosure increase liquidity. The optimal security design in these papers increases trading and liquidity by either making the security information-insensitive or decreasing information asymmetry. In contrast, asymmetric information about the fundamental value is less important in the market for agency MBS because the credit risk is guaranteed. While there may be information asymmetry about the value of the asset that TBA sellers are planning to sell, sellers cannot signal such in the TBA market. Furthermore, our results indicate that lenders' optimal pooling partially unravels the TBA market and decreases liquidity in the agency MBS market. We contribute to this literature in two ways. First, we show that trading structure may play a large role in determining the security design. Second, we also show whether the optimal choice

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<sup>3</sup>A few papers study other determinants of security designs. For instance, Hartman-Glaser, Piskorski, and Tchisti (2012) consider the optimal contract that incentivizes lenders to engage in costly hidden efforts to screen borrowers. Adelino, Frame, and Gerardi (2017) show that large investors of non-agency MBS influence which loans are pooled into securities they purchase.

of the security issuer is beneficial or damaging for trading and liquidity would depend on particular frictions and the market structure.

## 2 Institutional detail

An MBS is a pass-through security that is backed by a pool of mortgages. Agency MBS—which account for most of MBS—are typically backed by the GSEs (Freddie Mac and Fannie Mae) and Ginnie Mae, and hence, investors bear no credit risk. Thus, prepayment is the most important factor in agency MBS valuation as investors generally dislike prepayment.

Agency MBS can be sold in the TBA or SP markets.<sup>4</sup> TBA contracts are forward contracts, in which buyers and sellers agree only on six parameters (maturity, agency, price, settlement date, coupon, trade amount) at the time of the trade. As a result, TBA sellers have incentives to deliver the cheapest MBS that satisfies the contract terms—for example, one with a large average loan size. In contrast, buyers and sellers trade specific MBS in the SP market, and MBS-level prepayment characteristics can be priced in the SP market at a higher trading cost.

Thus MBS buyers and sellers face a trade-off between liquidity and adverse selection in deciding whether to trade in the TBA or SP market. MBS buyers pay lower trading costs in the TBA market, around 1 basis point (bp), but are also more likely to get delivered the cheapest MBS that meet the six TBA parameters. TBA contracts would be priced accordingly, regardless of the value of the specific MBS delivered. Hence, MBS sellers tend to trade MBS with high values in the SP market despite its higher trading cost of around 40 bps. We will document this empirically in Section 4.

Lenders have largely two ways of securitizing and selling mortgages. First, they may contribute new originations to a multi-lender MBS created by the GSEs or Ginnie Mae in exchange for cash or a slice of the multi-lender MBS. Smaller lenders tend to utilize this channel, but larger lenders often sell some portion of the mortgages in this way. Second, lenders may create pools themselves from loans that they originate or buy from smaller lenders, and “swap” the collection of mortgages for agency MBS. Lenders can then directly sell these MBS. MBS swap is usually done by larger lenders that originate enough mortgages each month. In the empirical section, we will mostly focus on MBS swap as GSEs or Ginnie Mae likely will not behave as price takers and may have different incentives from individual lenders.

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<sup>4</sup>Not all agency MBS are eligible to be delivered against TBA contracts. In this paper, we focus on loans that are in TBA-eligible MBS since we are interested in the effect that the choice between TBA and SP markets has on lenders’ pooling.

### 3 Data

We draw from multiple data sources. First, we use eMBS data to capture pooling patterns. Importantly, the loan-level data from eMBS links each loan to the MBS that it is pooled into. It also has other loan information such as loan size and location. We also pull security-level data such as issuance date, maturity, agency, TBA eligibility, and average and maximum loan size at issuance from eMBS.

Second, we use regulatory TRACE data to look at turnover and SP trade distribution. The regulatory version has a few advantages over the publicly available version for MBS data; the sample period goes back further, and we have actual trade volumes for larger trades. The latter turns out to be especially important because a large fraction of trades (in terms of volume) have truncated volume data in the publicly available version, and the exact volume information is useful for our purposes.

Third, we use data on System Open Market Account (SOMA) portfolio, which is the portfolio held by the Federal Reserve (or the Fed, henceforth), from the New York Fed’s website. We use this information to back out the CUSIPs and quantities of MBS that were delivered to the SOMA portfolio. Since the Fed exclusively traded through the TBA market during our sample period, an increase in face value for a given MBS held in the SOMA portfolio taking into account amortization and prepayments means that the MBS were delivered as TBA settlement. We assume that the characteristics of the MBS that was delivered to the Fed are roughly similar to those that are delivered to other TBA buyers and use the distribution of MBS characteristics that was delivered to the Fed as the distribution of TBA settlements. Because the Fed was a large buyer of agency MBS at that time, this is not a very unrealistic assumption.

Lastly, we use Fixed Income Data Feed from ICE Data Pricing & Reference Data to get monthly prices for MBS.<sup>5</sup> This dataset provides daily reference prices across a wide range of fixed-income securities, even for days in which there are no trades in the security. We pull month-end prices for all MBS and TBA contracts in our dataset.

For a majority of our empirical analyses, we focus on Fannie Mae and Freddie Mac MBS. MBS from these two agencies are more comparable to each other than Ginnie Mae MBS. Further, we only keep loans in TBA-eligible MBS in our sample because TBA-ineligible MBS cannot be traded in the TBA market. This means that we exclude adjustable-rate mortgages and other mortgages with non-standard features. For the most part, we only consider loans in 30-year MBS. Our sample period is from 2012 and 2015, which is the period when the financial market was relatively stable after the Global Financial Crisis. We do not include more recent years because preparations for UMBS might have impacted the market starting a few

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<sup>5</sup>This was formerly called Interactive Data Corporation Fixed Income Data Feed.

years before the UMBS introduction in 2019. For the tests in which we study lenders' pooling decisions, we further restrict the sample to MBS swaps because the agencies may have incentives other than profit maximization that drive their pooling decisions.

## 4 Empirical patterns

In this section, we document pooling and subsequent trading patterns to provide evidence that pooling decisions are influenced by the cheapest-to-deliver pricing in the TBA market and result in greater MBS heterogeneity. Throughout the paper, we focus on the loan size as a proxy for loan value since it influences expected prepayment speeds the most. It is also commonly used by market participants to characterize expected prepayments of MBS.<sup>6</sup> Larger loans have higher prepayments because borrowers with larger loans are more likely to refinance when mortgage rates decrease.<sup>7</sup> Thus, investors typically prefer MBS backed by smaller loans if all other characteristics are similar.

In our analysis, we will often use the maximum or average original balance of loans in MBS. Define MBS  $m$  as a collection of loans such that  $m = \{1, \dots, N_m\}$ , where  $N_m$  is the number of loans in the MBS. Denote the distribution of original loan balances within MBS  $m$  as  $B_m = \{b_1, \dots, b_{N_m}\}$ . The maximum original balance of loans in MBS  $m$  is

$$\max bal_m = \max(B_m). \quad (1)$$

The average original balance of loans in MBS  $m$  is

$$\text{avg} bal_m = \sum_{i=1}^{N_m} b_i / N_m. \quad (2)$$

**MBS pooling patterns and MBS heterogeneity** Figure 1 shows lenders' pooling patterns and their impact on MBS heterogeneity. Consider loan  $i$ , which is packaged in MBS  $m(i)$ . We plot the characteristics of MBS  $m(i)$  against the original balance of loan  $i$ . Figure 1(a) presents the share of loans in different  $\max bal$  bins against original loan balances.<sup>8</sup> It is obvious that pooling is far from random and changes discretely at

<sup>6</sup>For instance, see <https://cdn.ihs.com/www/blog/commentary/pdf/Markit-Agency-RMBS--Specified-Pool-Summary--December-2016.pdf>. There are other observable characteristics related to prepayments such as the LTV at origination, whether a loan is for an investment property, whether a loan is for a property in the state of New York, etc. However, market participants commonly use the loan size to characterize expected prepayments. During the sample period, MBS backed only by loans with smaller size (up to \$175,000), which have lower expected prepayments, account for 20% of agency MBS issuance. MBS only backed by loans with high LTV (greater than 80), which also have lower expected prepayments, account for 5%. Other MBS with lower prepayments have even smaller shares. Later we show that our empirical results are robust when we consider the LTV at origination.

<sup>7</sup>Figure A.4 in Appendix C shows prepayment speeds by loan size.

<sup>8</sup>The cutoffs for  $\max bal$  bins in this figure are those that are typically used in the industry.

different loan-size cutoffs. Smaller loans are more likely to be pooled with loans of similar sizes into the same MBS, which will have higher value because of their low ex-ante prepayments. For example, about 90% of loans not larger than \$85,000 are included in MBS that are only backed by loans not larger than \$85,000, and the rest are pooled into MBS that includes at least one very large loan with a size greater than \$400,000. This degree of pooling increases as the loan size increases, and there is a sharp change at \$175,000.

Figure 1(b) shows that the average of  $avgbal_m$  also jumps discretely at the same loan size cutoffs with the largest change at \$175,000. For loans with size larger than \$175,000,  $avgbal_m$  is rather flat. This figure also shows that larger loans tend to be included in MBS backed by greater numbers of loans with a sharp change at \$175,000. This finding suggests that smaller loans are more likely pooled separately into higher-value MBS, whereas lenders pool all other loans together to create a large MBS despite large heterogeneity in prepayment speeds among these loans. This pooling pattern is likely driven by the trade-off between adverse selection and liquidity in the TBA market. Lenders are able to sell MBS backed by smaller loans in the SP market at prices high enough to forego the liquidity benefit of the TBA market. However, as the loan size increases, the value of pooling loans with similar sizes decreases and is outweighed by the TBA liquidity benefit.

This pooling pattern results in greater MBS heterogeneity as shown in Figure 1(c), which will lead to more adverse selection in the TBA market. Although the loan-size distribution is relatively smooth up to around \$400,000, the distribution of  $avgbal_m$  is roughly bimodal, with a large group of MBS below \$175,000 and another group at around \$300,000.<sup>9</sup> The former group of MBS account for a sizeable share (20%) of agency MBS issuance during the sample period. If lenders pooled all loans together into a single MBS, the distribution of  $avgbal_m$  would have a single peak with limited heterogeneity across MBS.

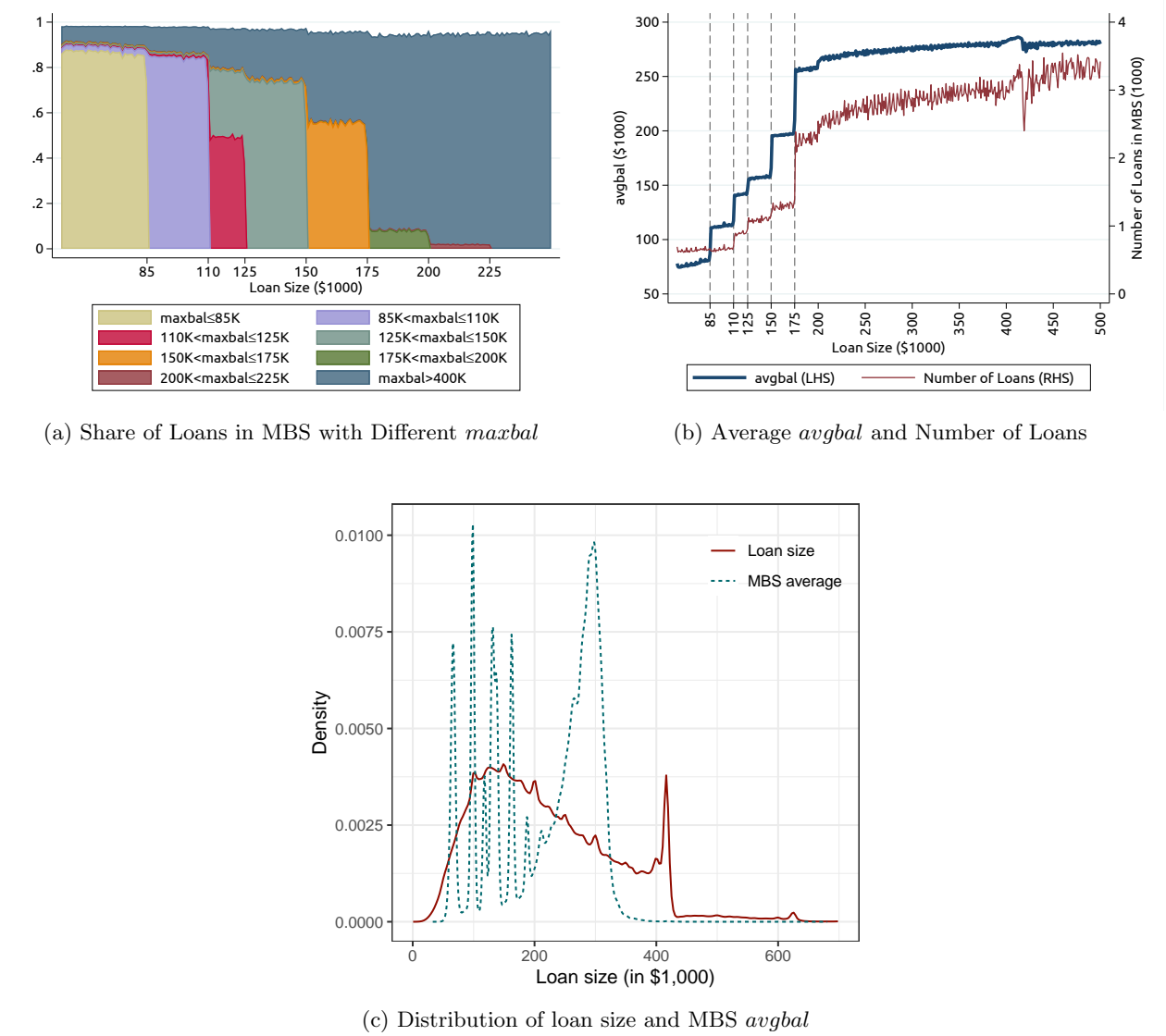
**Trading Patterns** Figure 2 plots the distribution of  $avgbal$  separately for TBA and SP trades.<sup>10</sup> This figure shows that MBS with low average loan size—in other words, high-value MBS—mostly trades in the SP market: only a tiny fraction of TBA trades are MBS with  $avgbal$  below \$175K. Hence, a vast majority of high-valued MBS trade in the SP market due to the cheapest-to-deliver pricing in the TBA market.<sup>11</sup> Note that the SP trading of MBS backed by smaller loans would not occur if a lender pooled such loans with larger loans. Also, there is still variation in  $avgbal$  of MBS that trade in the TBA market, suggesting that

<sup>9</sup>The peak just above \$400,000 in the loan size distribution is due to the bunching of loans just below the maximum loan size eligible for GSE securitization.

<sup>10</sup>Figure A.5 in the appendix presents the distribution of  $maxbal$  for TBA and SP trades, which shows qualitatively similar patterns to Figure 2.

<sup>11</sup>The figure also shows that MBS with  $avgbal$  above \$175,000 often trade in the SP market. Newly issued MBS backed by larger loans often trade in the SP market because it has relatively slower prepayment speeds than older MBS backed by larger loans. We do not focus on *exogenous* MBS heterogeneity like MBS ages in this paper.

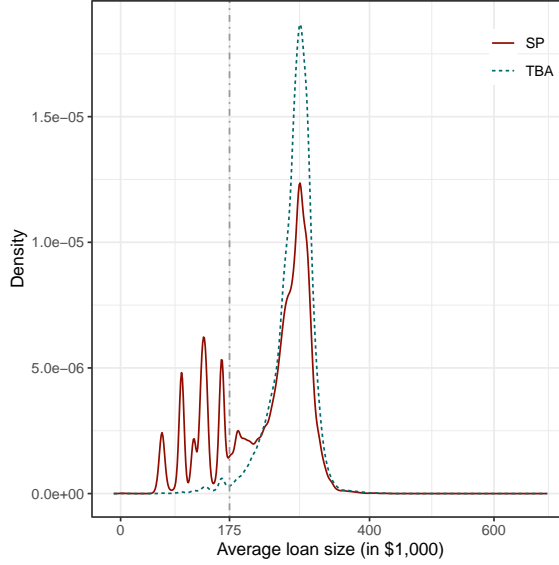
Figure 1: **MBS Pooling and MBS Heterogeneity:** Below figures describe the pooling pattern and MBS heterogeneity. Panels (a) and (b) present the relationship the loan size and the characteristics of MBS in which each loan is included. In both panels, the x-axis is for the loan balance bin with the size of \$1,000, and the y-axis is for characteristics of MBS in which each loan is packaged. Panel (a) displays the cumulative share of loans in MBS with different  $maxbal_m$ ; panel (b) displays the average of  $avgbal_m$  and  $N_m$ . Panel (c) plots the distribution of loan size and  $avgbal_m$ . For the distribution of  $avgbal_m$ , the MBS-level observation is weighted by the number of loans ( $N_m$ ) to make the two density plots comparable. Data source: eMBS.



the adverse selection problem in the TBA market is not too severe to the point where only the very worst MBS trades in the TBA market.

In summary, our findings suggest that the TBA trading structure leads to a trade-off between adverse selection and liquidity in lenders' pooling and trading decisions and that their strategies facilitate partial

Figure 2: **Distribution of  $avgbal$  for TBA and SP Trades:** The below figure plots the distribution of  $avgbal$  separately for TBA and SP trades. TBA trades are backed out from SOMA holdings data, and SP trade data is from the regulatory TRACE data. We weight the observations by trade volume. The vertical line is at \$175,000. Data Source: Federal Reserve Bank of New York and TRACE



unravelling of the TBA market. For high-valued loans, the cost from adverse selection in the TBA market appears greater than its liquidity benefit. Thus, lenders separate out high-value loans to create high-value MBS. This pooling strategy in turn results in even greater heterogeneity in MBS value and will exacerbate adverse selection in equilibrium, potentially making lenders as a whole worse off. In the next section, we build a model on MBS pooling and trading to understand how lenders' optimal pooling and trading strategies determine TBA price, liquidity, and adverse selection in equilibrium and contrast it with the social planner solution. We also study how policy changes affect the TBA market equilibrium.

## 5 Model

### 5.1 Setup

There are mass 1 of risk-neutral mortgage lenders in the economy. Lenders originate mortgages, package those mortgages into pools (MBS), and sell the pools in the secondary market. Each lender originates mortgages with fundamental values that are distributed uniformly in  $[0, a]$ . Individual lenders and mortgages are infinitesimally small.

The model has two stages. In the first stage, lender  $i$  chooses  $x_i \in [0, 1]$  and pools the bottom  $x_i$  share

of the loan distribution together—in other words, loans  $[0, ax_i]$ —into a single pool. In the second stage, he sells the pool in either the TBA or SP market. He also sells all other loans  $(ax_i, a]$  as separate loans—i.e., “pools” with zero mass. For the rest of the section, the word “pool” will refer to both the pools and ones that are sold as individual loans.

In the first stage, lender  $i$ ’s choice of  $x_i$  results in two types of pools: one pool with fundamental value  $\frac{ax_i}{2}$  and size  $x_i$ , which we will refer to as “cheapest-to-deliver pool”; and infinitesimally small pools that are distributed uniformly over  $(ax_i, a]$ . We refer to  $x$ , the share of loans that are pooled into the cheapest-to-deliver pool, as the “degree of pooling,” where a higher  $x$  reflects more pooling and less separation. It is sometimes more convenient to look at a lender’s pooling strategy in terms of  $1 - x$  instead of  $x$ . We will refer to  $1 - x$  as the “degree of separation.”  $x$  is also inversely related to MBS value heterogeneity: if all lenders choose  $x$ , the variance in MBS values is  $a^2(1 - x^3)/12$ . The lender pays an issuance cost  $K(x)$  if he chooses  $x$  as the degree of pooling.  $K(x)$  is bounded, decreasing and convex in  $x \in [0, 1]$ . With a higher degree of pooling, a lender has fewer pools to sell, decreasing operational costs. Lender  $i$  chooses  $x_i$  that maximizes the expected revenue from selling the pools in the second stage minus  $K(x)$ .

The pooling strategy allowed in our model abstracts away from details in actual pooling strategies for tractability. Yet it captures the main incentives that lenders face in their pooling decisions. As shown in Figure 1, there is a clear threshold at \$175,000 in the data, and loans above this threshold tend to be pooled together into large cheapest-to-deliver MBS. In reality, the \$175,000 threshold itself remains relatively constant by market convention, but the likelihood of including a higher-valued loan (i.e., a loan with size below the threshold) in a cheapest-to-deliver MBS (i.e., an MBS with maximum loan size above the threshold) changes depending on market conditions. If more higher-valued loans are included in cheapest-to-deliver MBS, then there will be fewer high-value MBS, as a higher  $x$  in our model does. Moreover, what matters most is how lenders’ pooling strategies affect heterogeneity in MBS value, which in turn determines adverse selection in the TBA market. Increasing the share of high-value loans in large cheapest-to-deliver MBS decreases heterogeneity in MBS values, as higher  $x$  in our model does.

In the second stage, we assume that there is an infinite mass of risk-neutral buyers in both the TBA and SP markets that are willing to buy the pools at the expected fundamental values. Sellers have to pay a transaction cost  $b_0 - m(c_0 + c_1q)$  to market makers in the TBA market and  $b_0 + (1 - m)(c_0 + c_1q) + \epsilon$  in the SP market.  $\epsilon$  is independent across trades and is distributed  $f(\epsilon) = \text{unif}[0, \bar{c}]$ , and  $q$  is the TBA market trading volume. Since SP volume is  $1 - q$ , TBA and SP trading costs decrease with the trading volume in the respective markets. This feedback is likely true in the data since liquidity, trading costs, and trading

volume are all highly related. Lastly,  $m$  is a parameter that takes on a value between 0 and 1. For a lender's decision of whether to sell in the TBA or SP market, however, only the difference between SP and TBA trading costs ( $c_0 + c_1q + \epsilon$ ) matters, regardless of the value of  $m$ .<sup>12</sup>  $m$  determines how much the trading cost difference is attributable to a lower TBA trading cost rather than a higher SP trading cost.

With this trading cost specification, we implicitly assume that sellers have to pay a higher transaction cost in the SP market, which reflects that providing liquidity in a less-liquid SP market requires higher dealer compensation.<sup>13</sup> The first term of the trading cost difference,  $c_0$ , is the minimum cost difference. The second term,  $c_1q$ , makes the trading cost difference increase with higher TBA trading volume, or equivalently, with lower SP trading volume. Thus, lenders' pooling and trading decisions in aggregate affect the trading cost difference in equilibrium. The last term,  $\epsilon$ , is the stochastic component, which is realized after the first stage but before choosing between the TBA and SP markets. Thus, a lender's pooling decision does not depend on the realized value of  $\epsilon$ , but his trading decision does. A stochastic  $\epsilon$  term and the assumption on its distribution ensures that equilibrium exists. To ensure existence and uniqueness of the equilibrium, we assume  $c_0, c_1, m \geq 0$ ,  $c_0 + c_1 \leq \frac{1}{2}a$ , and  $\bar{c} > c_1 + a$ .

For a pool with fundamental value  $v$ , lenders can sell the pool in the TBA market for  $p - b_0 + m(c_0 + c_1q)$  and in the SP market for  $v - \epsilon - b_0 - (1 - m)(c_0 + c_1q)$ .  $p$  is the TBA market price and does not directly depend on the fundamental value of the pool. Thus, a lender sells pool  $j$  with fundamental value  $v$  in the TBA market if and only if  $p > v - \epsilon_j - c_0 - c_1q$ . Hence, this pool will be sold in the TBA market with probability  $1 - F(v - p - c_0 - c_1q)$ . There is an infinite mass of risk-neutral buyers in the TBA market, so the TBA price  $p$  would equal the expected fundamental value of MBS that are traded via TBA.

## 5.2 Second Stage Equilibrium

We first consider the equilibrium in the second stage, where lenders sell each pool in either the TBA or SP market.  $p(x)$  and  $q(x)$  denote the equilibrium  $p$  and  $q$  when all lenders choose  $x$  as the degree of pooling in the first stage of the model. Given the assumptions,  $p(x)$  and  $q(x)$  are the  $p$  and  $q$  that satisfy the following

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<sup>12</sup>For this reason, without loss of generality, we can set TBA trading costs as zero and SP trading costs as  $c_0 + c_1q$ . This does not change any of the model results. We choose the current specification instead to make it clear that TBA trading costs and SP trading costs may both depend on  $q$ .

<sup>13</sup>This assumption reflects the fact that individual CUSIPs are thinly traded in the SP market.

system of equations.

$$p = \frac{\frac{ax^2}{2} \left(1 - F\left(\frac{ax}{2} - c_0 - c_1q - p\right)\right) + \int_{ax}^a \frac{v}{a} \left(1 - F(v - c_0 - c_1q - p)\right) dv}{x \left(1 - F\left(\frac{ax}{2} - c_0 - c_1q - p\right)\right) + \int_{ax}^a \frac{1}{a} \left(1 - F(v - c_0 - c_1q - p)\right) dv}, \quad (3)$$

$$q = x \left(1 - F\left(\frac{ax}{2} - c_0 - c_1q - p\right)\right) + \int_{ax}^a \frac{1}{a} \left(1 - F(v - c_0 - c_1q - p)\right) dv \quad (4)$$

where  $F(\epsilon)$  is the cumulative distribution function of  $\epsilon$ .

Because we assume that all lenders choose the same  $x$ , TBA price is not lower than the value of the cheapest-to-deliver pool ( $\frac{ax}{2}$ ), and, thus, the cheapest-to-deliver pool is always traded through the TBA market as stated in the following proposition. All proofs are relegated to Appendix A.

**Proposition 1.** *If all lenders choose the same  $x \in [0, 1]$ , all of the cheapest-to-deliver pool is sold in the TBA market.*

We first show the existence and uniqueness of the equilibrium TBA price and volume (Theorem 1).

**Theorem 1.** *If all lenders choose the same  $x \in [0, 1]$ , there exists a unique TBA price  $p(x) \in [\frac{1}{2}x, 1]$  and a unique TBA volume  $q(x) \in [x, 1]$ .*

We are interested in how the TBA price  $p(x)$  and volume  $q(x)$ , as well as the degree of adverse selection, behave in equilibrium. Since trading volume and liquidity are positively correlated empirically, and since the SP/TBA trading cost difference increases with TBA volume in our model, we can think of TBA volume  $q$  as a proxy for TBA market liquidity. The degree of adverse selection in the TBA market can be gauged by looking at the average value of MBS delivered through the TBA market relative to the average value of MBS. More precisely, we measure adverse selection as  $s(x; c, a) = 1 - \frac{p(x)}{a}$ , where higher values would correspond to greater adverse selection. We use  $c$  as a shorthand for  $c_0$ ,  $c_1$ , and  $\bar{c}$ , and higher  $c$  values correspond to a higher SP/TBA trading cost difference. While  $p(x)$  and  $s(x)$  move in opposite directions, we look at them separately because we are also interested in how the equilibrium changes with loan dispersion  $a$ . We first explore how these equilibrium outcomes vary with  $x$  in Proposition 2.

**Proposition 2.**  *$p(x)$  and  $q(x)$  are weakly increasing in  $x \in [0, 1]$ , and strictly increasing in  $x$  if  $x > \frac{1}{a}(c_0 + c_1q(0) + p(0))$ .  $s(x)$  is weakly decreasing in  $x \in [0, 1]$ , and strictly decreasing in  $x$  if  $x > \frac{1}{a}(c_0 + c_1q(0) + p(0))$ .*

Proposition 2 shows that a higher degree of pooling (a higher  $x$ ) leads to higher TBA price, TBA volume, and lower adverse selection. A higher degree of pooling leads to lower MBS heterogeneity and the cheapest-to-deliver pool having a higher value. Because this pool is always traded in the TBA market, a higher  $x$

increases the expected value of MBS delivered in the TBA market. Then sellers trade more higher-valued pools through the TBA market, raising the expected value even higher. In turn, there is an additional feedback effect through reduced TBA trading cost due to higher TBA volume. As a result,  $p(x)$  and  $q(x)$  increase in  $x$ , and  $s(x)$  decreases in  $x$ .

We now look at how TBA market liquidity and adverse selection changes with the SP/TBA trading cost difference ( $c$  parameters) and loan value dispersion ( $a$ ) when  $x$  is fixed.

**Proposition 3.** *For a given  $x < 1$ , TBA volume,  $q(x; c, a)$  strictly decreases with  $a$ , and strictly increases with  $c_0$ ,  $c_1$ , and  $\bar{c}$ . Also, under  $x < 1$ , TBA market adverse selection,  $s(x; c, a)$  strictly increases with  $a$ , and strictly decreases with  $c_0$ ,  $c_1$ , and  $\bar{c}$ . If  $x = 1$ ,  $q(x; c, a)$  and  $s(x; c, a)$  are constant.*

Holding the degree of pooling fixed, an increase in  $a$  or a decrease in  $c$  leads to an increase in MBS heterogeneity relative to the SP/TBA trading cost difference. As a result, adverse selection in the TBA market increases and TBA volume decreases as long as there exists heterogeneity in MBS value ( $x < 1$ ). If all loans are pooled into a single homogeneous MBS ( $x=1$ ), there is no scope for adverse selection regardless of trading costs or loan value distribution.

### 5.3 First Stage

We now look at how a lender optimally chooses the degree of pooling ( $x$ ) in the first stage given TBA price and volume. We define  $r(v; p, q)$  as the expected per-unit revenue that the lender receives from selling a pool with fundamental value  $v$  taking TBA market price  $p$  and volume  $q$  as given.

$$r(v; p, q) = p(1 - F(v - c_0 - c_1q - p)) + (v - c_0 - c_1q)F(v - c_0 - c_1q - p) - \int_0^{v - c_0 - c_1q - p} \epsilon f(\epsilon) d\epsilon - (b_0 - m(c_0 + c_1q)) \quad (5)$$

Define  $\pi(x; p, q) = R(x; p, q) - K(x)$  to be the total payoff to the lender that chooses  $x$  given  $p$  and  $q$ .  $R$  is the lender's total revenue. Precisely,

$$R(x; p, q) = xr\left(\frac{ax}{2}; p, q\right) + \int_{ax}^a \frac{1}{a} r(v; p, q) dv.$$

$K$  is the issuance cost, which we assume to be bounded, decreasing, and convex in  $x \in [0, 1]$ . With a higher  $x$ , a lender has fewer pools to sell, decreasing operational costs.

Since we consider the competitive equilibrium, each lender chooses  $x$  that maximizes  $\pi(x; p, q)$ , taking TBA price  $p$  and volume  $q$  as given. In choosing the optimal  $x$ , a lender trades off between the issuance cost and trading flexibility. On the one hand, the issuance cost ( $K$ ) is smaller for higher  $x$ . On the other hand, lower  $x_i$  allows for greater flexibility in choosing between the TBA and SP markets for each pool. The lender's revenue  $R(x_i; p, q)$  is weakly increasing if  $x_i$  decreases because a lower degree of pooling provides greater trading flexibility for more loans. If a loan is pooled with other loans into the same MBS, the trading decision in the second stage is made based on the average value of the MBS, not the fundamental value of individual loans. Thus, trading loans with higher fundamental values separately increases the trading revenue. This trading flexibility benefit is smaller for loans with lower fundamental value because they would be less frequently traded in the SP market. Therefore, given the issuance cost, the lender always pool some loans together into the same MBS by choosing a positive  $x_i$ .

The following theorem shows that given TBA price and volume ( $p$  and  $q$ ), there exists a unique optimal degree of pooling for an individual lender. Note that  $p$  and  $q$  are determined by all other lenders' choices of the degree of pooling, and an individual lender takes them as given. Therefore, this theorem is about the individual lender's problem, not the equilibrium.

**Theorem 2.** *Given market TBA price  $p$  and quantity  $q$ , there exists a unique  $x(p, q) \in (0, 1]$  that maximizes  $\pi(x; p, q)$ .*

## 5.4 Equilibrium and Social Planner Problem

We focus on symmetric competitive equilibrium in which all lenders choose the same  $x$ . We first show that such equilibrium exists and is unique given our assumptions and that the unique equilibrium is interior as long as the issuance cost does not decrease too rapidly for  $x$  near 1.

**Theorem 3.** *There exists a unique symmetric competitive equilibrium in which all originators choose the same  $x \in (0, 1]$ . Additionally, if*

$$K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}},$$

*the equilibrium  $X$  is such that  $X < 1$ . Otherwise,  $X = 1$ .*

We contrast the competitive equilibrium with the social planner solution. The social planner chooses  $z$ , the degree of pooling, for all originators, which determines  $p(z)$ , the TBA price, and  $q(z)$ , the TBA volume. His objective is to maximize

$$L(z) = R(z; p(z), q(z)) - K(z).$$

Note that he takes into account the effect of  $z$  on the TBA price and volume ( $p(z)$  and  $q(z)$ ) in contrast to an individual lender, who takes them as given in choosing the optimal pooling (in Theorem 2). This difference in incentives leads to a stark difference in the optimal pooling between the planner and a lender.

**Theorem 4.** *The social planner always chooses  $z = 1$  as the optimal degree of pooling.*

Thus, the degree of pooling in a competitive equilibrium is always lower than the social planner solution, as long as the issuance cost does not decrease too rapidly for  $x$  near 1. Given  $p$  and  $q$ , choosing a lower  $x$  gives the lender more freedom to optimally choose between TBA and SP markets. However, this facilitates partial unravelling of the TBA market in equilibrium. If every lender chooses lower  $x$ , the TBA price  $p$  is also lower in equilibrium, which in turn results in lower TBA trading volume and higher TBA trading cost. This feedback effect further induces lenders to choose even a lower  $x$ . Individual lenders do not take into account this externality they have on the TBA price and volume by choosing a low  $x$ , whereas the social planner does. Therefore, the social planner chooses a higher degree of pooling than the lender. In fact, the planner chooses to pool all loans into a single MBS and sell all of them in the TBA market to minimize the total transaction cost.<sup>14</sup>

## 5.5 Comparative Statics

We study how the equilibrium pooling decision, the degree of adverse selection in the TBA market, and the TBA volume change with two sets of parameters. First is  $c_0$ ,  $c_1$ , and  $\bar{c}$ , which determine the trading cost difference between the SP and TBA markets. We consider changing one of the three trading cost parameters leaving the other two fixed.<sup>15</sup> Second is loan value dispersion  $a$ .

Define  $X(c, a) = X(c_0, c_1, \bar{c}, a)$  as the competitive symmetric equilibrium  $x^*$  under parameters  $c_0$ ,  $c_1$ ,  $\bar{c}$ , and  $a$ . We use  $c$  as a shorthand for  $c_0$ ,  $c_1$ , or  $\bar{c}$ . The equilibrium TBA price under parameters  $c$  and  $a$  is defined as  $P(c, a) = P(X(c, a); c, a)$ . Similarly, we define  $Q(c, a)$  and  $S(c, a)$  as the equilibrium TBA volume and the degree of adverse selection, respectively, under parameters  $c$  and  $a$ .

Proposition 4 shows how the equilibrium degree of pooling changes with trading cost parameters and loan dispersion.

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<sup>14</sup>Pooling all loans into a single MBS would make the planner indifferent between trading in the TBA and SP markets in a model where the transaction cost is more micro-founded because the two markets will be the same, resulting in the same transaction costs of the two markets. However, the key intuition of our result is that the planner would optimally pool all loans to minimize the transaction cost in trading. This would be also true in a more micro-founded model.

<sup>15</sup>Since we assume  $\bar{c} > c_1 + 1$ , our model does not allow all  $c$  parameters to go to zero—in other words, SP trading costs will always be strictly higher than TBA trading costs. Because an individual security will be always thinly traded in the SP market, this assumption is reasonable.

**Proposition 4.**  $X(c, a)$  is weakly increasing in  $c_0$ ,  $c_1$ , and  $\bar{c}$  and weakly decreasing in  $a$ . Moreover,  $X(c, a)$  is strictly increasing in  $c_0$ ,  $c_1$ , and  $\bar{c}$  and strictly decreasing in  $a$  if  $K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}}$ .

We then look at how TBA volume and adverse selection change with these parameters.

**Proposition 5.**

$$\begin{aligned} \frac{dS(c, a)}{dc} &\leq \frac{\partial s(X; c, a)}{\partial c} \leq 0, & \frac{dS(c, a)}{da} &\geq \frac{\partial s(X; c, a)}{\partial a} \geq 0 \\ \frac{dQ(c, a)}{dc} &\geq \frac{\partial q(X; c, a)}{\partial c} \geq 0, & \frac{dQ(c, a)}{da} &\leq \frac{\partial q(X; c, a)}{\partial a} \leq 0 \end{aligned}$$

That is, adverse selection  $S(c, a)$  decreases (increases) with  $c$  ( $a$ ), and TBA volume  $Q(c, a)$  increases (decreases) with  $c$  ( $a$ ). Moreover, changes in the equilibrium degree of pooling  $X(c, a)$  lead to greater changes to the TBA market adverse selection and volume than when the degree of pooling is fixed. All inequalities hold with strict inequalities if  $K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}}$ .

Proposition 5 shows that changing  $c$  or  $a$  impacts the TBA market adverse selection and volume in two ways. First, in the partial equilibrium setting in which the degree of pooling is fixed, an increase in trading cost or a decrease in loan dispersion increases TBA liquidity and decreases adverse selection if there exists heterogeneity in MBS value. This result was already mentioned in Proposition 3. Second, in the full equilibrium in which the degree of pooling is chosen optimally, the change in the degree of pooling would affect MBS value heterogeneity itself, which would further impact TBA price, volume, and adverse selection.

## 5.6 Discussion

**Policy Implications** Our theoretical results have important policy implications. First, our results on loan value dispersion have implications for housing finance reforms. Ever since the GSEs entered conservatorship in 2008, numerous proposals for reforming the housing finance market have been presented, and keeping the TBA market liquid is an important consideration for policymakers and practitioners alike.<sup>16</sup> Therefore, it is important to evaluate how each proposal would impact TBA market liquidity. For instance, one of the popular proposals is to leave the two GSEs largely in their current form but privately recapitalize them. If there is no government guarantee, or if there is some ambiguity about government guarantee, loan value

<sup>16</sup>For instance, see the white paper “A Vision for Enduring Housing Finance Reform” by the National Association of Realtors, <https://www.nar.realtor/sites/default/files/documents/2019-Working-Paper-A-Vision-For-Enduring-Housing-Finance-Reform-02-07-2019.pdf>.

dispersion will increase, decreasing TBA market liquidity.<sup>17</sup> Another proposal is to increase competition and have multiple private guarantors in addition to the two GSEs. If the market perceives the value of guarantees from different guarantors to be different, this would have a similar effect to increasing loan value dispersion.

Second, our results on loan value dispersion also have implications for Uniform MBS (UMBS). The recent introduction of UMBS made Fannie Mae and Freddie Mac MBS fungible for TBA trading, which could increase loan value dispersion if loans securitized through the two GSEs have different prepayment speeds.

Third, our results on the SP/TBA trading cost difference suggest that reducing SP trading cost—for example, through post-trade transparency—is not necessarily desirable for the entire MBS market. Lower SP trading costs may impair TBA market liquidity. Moreover, an additional theoretical result (Proposition 6 in Appendix A) shows that it may even reduce lenders’ revenues and profits, which could result in higher mortgage rates for individual mortgage borrowers.

**Exogenous Heterogeneity in MBS Value** Our model focuses on endogenous heterogeneity of MBS value but abstracts away from exogenous sources of heterogeneity—for example, due to MBS ages. In the model, we assume that all MBS trade exactly once, in the month that the MBS is issued. In reality, MBS issued in different months are deliverable against the same TBA contract as long as the agency, coupon, and term match. Prepayment tends to be lower for newly issued loans and for fairly old loans, making the newest and oldest MBS more likely to trade through the SP market. This exogenous heterogeneity in MBS values is not affected by lenders’ pooling decisions. Although we abstract away from the exogenous heterogeneity, this assumption is not unreasonable because a majority of MBS traded in a given month are newly issued MBS.

For the rest of the paper, we empirically test the predictions of our model. First, we study how a lender’s pooling changes with loan value dispersion and the difference in the TBA/SP trading cost (Proposition 4) in Section 6. Second, we test predictions on adverse selection in the TBA market (Propositions 2 and 5) in Section 7.

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<sup>17</sup>The Securities Industry and Financial Markets Association expressed a similar concern about a potential move to privatize the GSEs prematurely before the end of the Trump administration. <https://www.sifma.org/resources/submissions/treasury-engagement-on-gse-conservatorships/>

## 6 Empirical Tests of Comparative Statics on MBS Pooling

In this section, we empirically test Proposition 4, which is a comparative statics result that predicts that if the SP/TBA trading cost difference increases or loan value dispersion narrows, lenders choose a higher degree of pooling. We test the prediction regarding loan value dispersion by comparing MBS with different coupons and by looking around the taper tantrum event. We test the prediction regarding SP/TBA trading cost difference by comparing Fannie Mae and Freddie Mac MBS.

In the empirical analysis, we use a “degree of separation” measure, which is inversely related to the degree of pooling and can be thought of as  $(1 - x)$ . Motivated by the pooling patterns shown in Figure 1, we measure the degree of separation with the probability that a loan with an original balance below \$175,000 (“low-balance loan”) is packaged into MBS with the maximum loan balance up to \$175,000 (“low-balance MBS”). We use the \$175,000 threshold because pooling patterns change sharply at this threshold, as evident in Figure 1. In the model, we assume for tractability that lenders choose the  $x$  threshold value and that loans above that threshold are sold separately; in reality, \$175,000 threshold itself remains relatively constant by market convention, but the likelihood of pooling loans with size below this threshold separately into low-balance MBS changes. Although the measures of the degree of separation in the model and in the data are not identical, the two measures still capture similar incentives that lenders face and move MBS value heterogeneity in a similar way, as discussed in Section 5.

### 6.1 Empirical Predictions

We first list the empirical predictions for each of the three tests.

**Different MBS Coupons** We compare pooling patterns across different MBS coupons within the same agency to test the empirical prediction that if loan value dispersion narrows, lenders choose a higher degree of pooling or, equivalently, a lower degree of separation.

All else equal, values for loans in MBS with higher coupons have wider dispersion than those in MBS with lower coupons. Loan value dispersion is higher for loans with higher interest rates because refinancing incentives tend to vary more among such loans. Conditional on the loan interest rate, large loans typically have higher prepayments than small loans. Because the fixed cost of refinancing limits the benefit of refinancing for small loans, refinancing incentives of large loans increase much faster with loan interest rates than those of small loans. As a result, the difference in prepayments between small and large loans is typically larger for loans with higher interest rates. Thus, the loan value dispersion is also wider for loans in MBS with higher

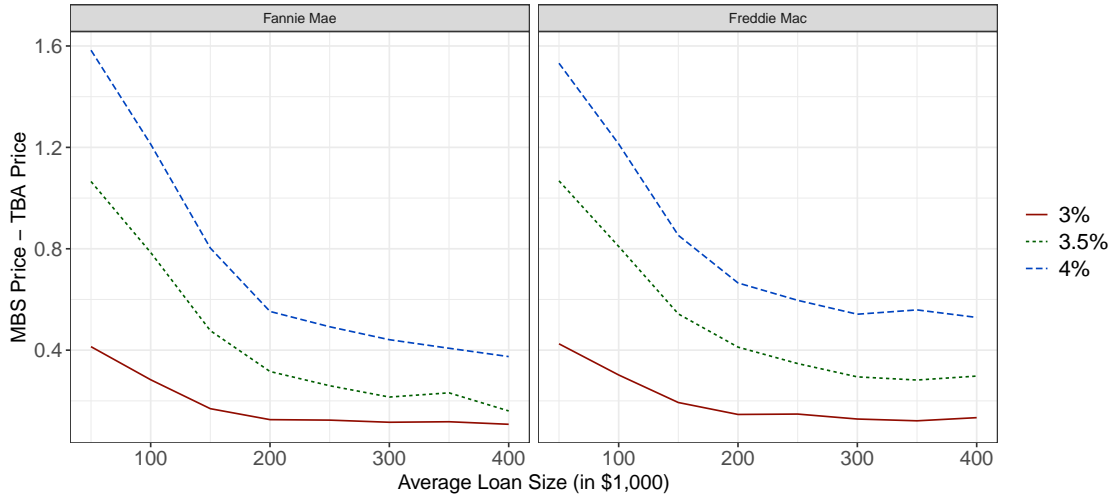
coupons.

Figure 3 shows that prices of MBS with higher coupons net of the corresponding TBA prices are more dispersed across MBS with different average loan size (*avgbal*). We take the difference between the quoted price for MBS and the corresponding TBA price to remove the time variation in MBS prices due to changes in prevailing mortgage rates or other market factors.<sup>18</sup> This figure supports that within the same agency and maturity, loans in MBS with lower coupons tend to have narrower fundamental value dispersion.<sup>19</sup>

Since Proposition 4 predicts that lenders choose a higher degree of pooling for narrower loan value dispersion, the model would predict that lenders choose a lower degree of separation (or a higher degree of pooling) for lower coupons. We compare pooling patterns between coupons with the largest issuance amounts (3%, 3.5%, and 4% coupons) during the sample period (2012–2015).

**Figure 3: Price dispersion and MBS coupon rates**

This figure plots the average difference between prices of MBS and their corresponding TBAs against the average loan size of the MBS (*avgbal*), separately by MBS coupon rates. We control for the LTV and the difference between the original weighted average coupon (WAC) and the MBS coupon rate by first regressing the price difference (MBS price minus TBA price) on LTV bins and (WAC – MBS coupon) and taking the average residual by *avgbal* bins. We add the average price difference for each MBS coupon since the average price difference varies with MBS coupons. Source: ICE Data Pricing & Reference Data, LLC.



**Taper Tantrum** Next, we use the 2013 taper tantrum (TT) event as another test for the empirical prediction on the change in loan value dispersion. From late May to late June of 2013, U.S. Treasury

<sup>18</sup>We provide detail about this figure in Appendix B.1.1.

<sup>19</sup>Because we do not have information about fundamental values of individual loans, we make two approximations: we use MBS-level data (so the x-axis is the average loan size instead of size), and we use prices instead of fundamental values. MBS prices include the value added due to TBA eligibility. This result, however, is sufficient to show that loans in MBS with lower coupons have narrower fundamental value dispersion.

yields increased dramatically over the fears of a tighter Federal Reserve policy. As a consequence, prevailing mortgage rates also increased fairly drastically during this time, leading to lower expected prepayments for a given MBS coupon rate. Figure 4 plots the prevailing mortgage rate during 2013, where the shaded area starts on May 22, 2013, when then-Fed Chairman Bernanke mentioned a potential plan for a taper before the Congress. Mortgage rates increased rapidly between late May and late June before eventually stabilizing at about 100 bps higher than the pre-TT level by July.

**Figure 4: Prevailing mortgage rates** The following graph plots the prevailing mortgage rate for 15- and 30-year fixed-rate mortgages. The shaded area is from May 22, 2013 through June 30, 2013. Source: Freddie Mac Primary Mortgage Market Survey



This sudden, sharp increase in mortgage rates reduced incentives to refinance after TT and resulted in less dispersed loan values. In Appendix B.1.2, we provide evidence that the loan value distribution did indeed shrink after TT. Proposition 4 predicts that narrower loan value dispersion would increase the degree of pooling, or in other words, that the degree of separation would decrease after TT.

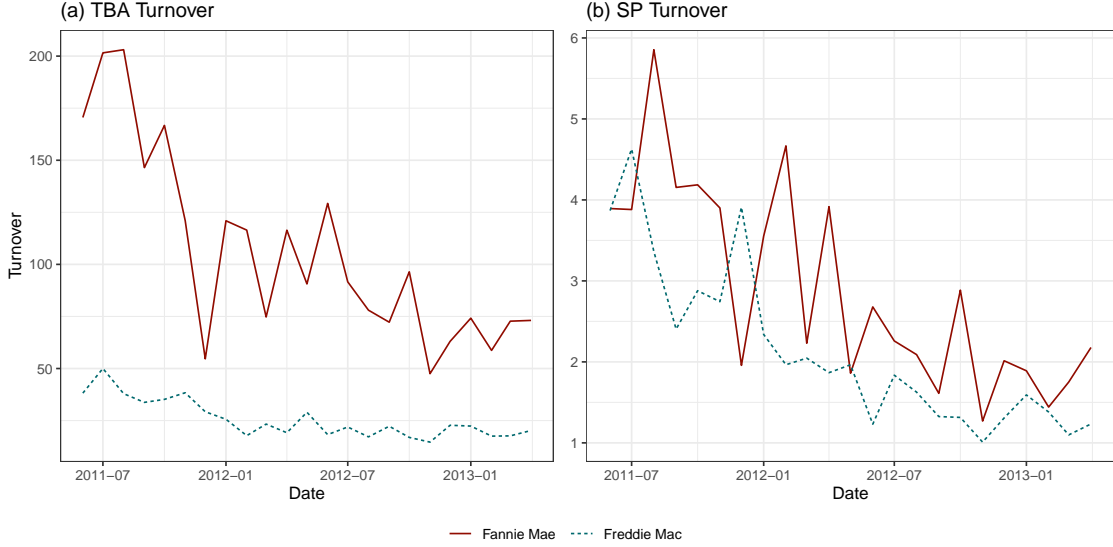
**Fannie Mae vs Freddie Mac** Lastly, we compare pooling patterns between Fannie Mae and Freddie Mac MBS to show that if the SP/TBA trading cost difference is higher, lenders choose a higher degree of pooling. Before the introduction of UMBS, Fannie Mae TBAs were typically much more liquid compared to Freddie Mac TBAs—in fact, UMBS was implemented because of the taxpayer cost associated with the difference between Fannie Mae and Freddie Mac TBA market liquidity.<sup>20</sup> Accordingly, Figure 5 shows that the turnover for Fannie Mae TBAs was significantly higher than that of Freddie Mac TBAs, implying higher

<sup>20</sup>See FHFA's request for input for single security structure for more detail: <https://www.fhfa.gov/PolicyProgramsResearch/Policy/Documents/RFI-Single-Security-FINAL-8-11-2014.pdf>

liquidity for Fannie Mae TBAs. In contrast, SP turnover is similar for Fannie Mae and Freddie Mac MBS.<sup>21</sup> Since trading volume and liquidity are highly correlated, these results imply that the difference between SP and TBA trading cost is higher for Fannie Mae MBS than for Freddie Mac MBS.

**Figure 5: Turnover for Fannie Mae and Freddie Mac MBS**

We plot the monthly turnover for TBA and SP markets in Panels (a) and (b), respectively. Turnover is plotted separately for Fannie Mae and Freddie Mac MBS. Calculation of turnover is described in the main text. Source: TRACE



Therefore, our model predicts that Fannie Mae MBS would have a higher degree of pooling, or in other words, that Freddie Mac MBS would have a higher degree of separation. We compare the pooling patterns between Fannie Mae MBS and Freddie Mac MBS during the sample period from 2012 to 2015.

## 6.2 Regression Specification and Results

For the three tests, we estimate the following regression.

$$1[\max bal_{m(i)} \leq 175K] = X_i(\beta) + z_i\gamma + \xi_i + \epsilon_i \quad (6)$$

The unit of analysis is each loan  $i$ , and  $m(i)$  refers to MBS in which loan  $i$  is packaged. The dependent variable is a dummy variable that is equal to 1 if the low-balance loan is packaged into a low-balance MBS. We estimate Equation (6) with low-balance loans only because the dependent variable would be zero by

<sup>21</sup>For the denominator in both TBA and SP turnover calculations, we use the monthly issuance amount of TBA-eligible MBS. TBA volume tends to correlate highly with recent issuance amount. For SP turnover numerator, we use trades of TBA-eligible MBS.

definition for all other loans. The dependent variable captures the probability that a low-balance loan ends up in a low-balance MBS, which is our proxy for the degree of separation. The estimation sample only includes loans that are securitized through MBS swap because the agencies may have different incentives from individual mortgage lenders.

On the right-hand side,  $X_i$  is the main independent variable, which varies depending on tests. First, for the comparison across MBS coupons,

$$X_i(\beta) = \sum_{c \in \{3.5, 4\}} 1[Coupon_i = c]\beta_c. \quad (7)$$

In this test, the omitted category is the loan included in MBS with a 3% coupon. The fixed effect  $\xi_i$  is at the level of origination year-month, state, agency, lender, and loan-size bin.

Second, for the comparison between before and after the TT event,

$$X_i(\beta) = 1[OriginationMonth_i \geq 2013:m5]\beta \quad (8)$$

where  $OriginationMonth_i$  is the loan origination month of loan  $i$ . We view that TT affected a lender's pooling strategy for loans originated since May 2013 because a loan is typically included in MBS that is issued one or two months after its origination month. In this test, the fixed effect  $\xi_i$  is at the level of state, coupon, agency, lender, and loan-size bin.

Third, for the comparison between Fannie Mae and Freddie Mac MBS,

$$X_i(\beta) = 1[Agency_i = Freddie\ Mac]\beta. \quad (9)$$

In this test, the fixed effect  $\xi_i$  is at the level of origination year-month, state, coupon, lender, and loan-size bin. In all of the three tests, we control for other characteristics  $z_i$ , which include credit score, loan-to-value ratio, debt-to-income ratio, owner-occupancy status, property type, and whether a loan is a purchase or refinance loan.

Table 1 displays the regression estimates for loans included in 30-year MBS. The results are consistent with the model predictions in all of the three tests. Low-balance loans in MBS with coupons of 3.5% and 4% are more likely to be included in low-balance MBS by 29.7 pp and 40.7 pp compared to 3% MBS. The probability that a low-balance loan is included in low-balance MBS dropped by about 19 pp after the TT event. This probability is larger for Fannie Mae MBS by 3.6 pp.

Given the unconditional probability of a low-balance loan to be included in a low-balance MBS of 75 percent, these estimates, especially those for the first two tests, are economically large. These results suggest that lenders' pooling practices are quite responsive to changes in incentives. Moreover, to the extent that incentives to increase the degree of separation would most likely prevail at times where TBA market adverse selection is already high, the optimal behavior of lenders may have large negative externalities, decreasing TBA market liquidity and making the adverse selection problem more severe during those times. We will study how lenders' pooling decisions impact adverse selection in the TBA market in Section 7.

**Table 1: Testing for Degree of Separation (30-year MBS):** This table presents the coefficient estimates for the regression in Equation (6). We estimate the regression using loans included in 30-year MBS. Column (1) compares the degree of separation across MBS with coupons of 3%, 3.5%, and 4%. Column (2) compares the degree of separation between periods before and after the TT event. Column (3) compares the degree of separation between Fannie Mae and Freddie Mac MBS. Standard errors are clustered at the same level as the fixed effect. The stars indicate: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ . Data source: eMBS.

	(1)	(2)	(3)
Coupons:			
1[Coupon=3.5%]	0.297*** (0.002)		
1[Coupon=4%]	0.407*** (0.002)		
Taper Tantrum:			
1[OriginationMonth $\geq$ 2013:m5]		-0.191*** (0.003)	
Fannie v Freddie:			
1[Agency=Freddie]			0.036*** (0.001)
Agency x Time x State x Lender x Loan Size Bin	Y		
Coupon x Agency x State x Lender x Loan Size Bin		Y	
Coupon x Time x State x Lender x Loan Size Bin			Y
Other Controls	Y	Y	Y
N. Obs.	2,282,157	486,058	1,621,161
Adj. $R^2$	0.60	0.61	0.64

## 6.3 Robustness Checks

We conduct further analyses to see whether our main results are robust to different estimation samples or different regression specifications.

### 6.3.1 15-year MBS

We estimate the same regression using loans included in 15-year MBS and report the estimates in Table A.3 of Appendix D. The results are also consistent with the model predictions and economically significant.

### 6.3.2 Monthly Changes in the Degree of Separation around Taper Tantrum

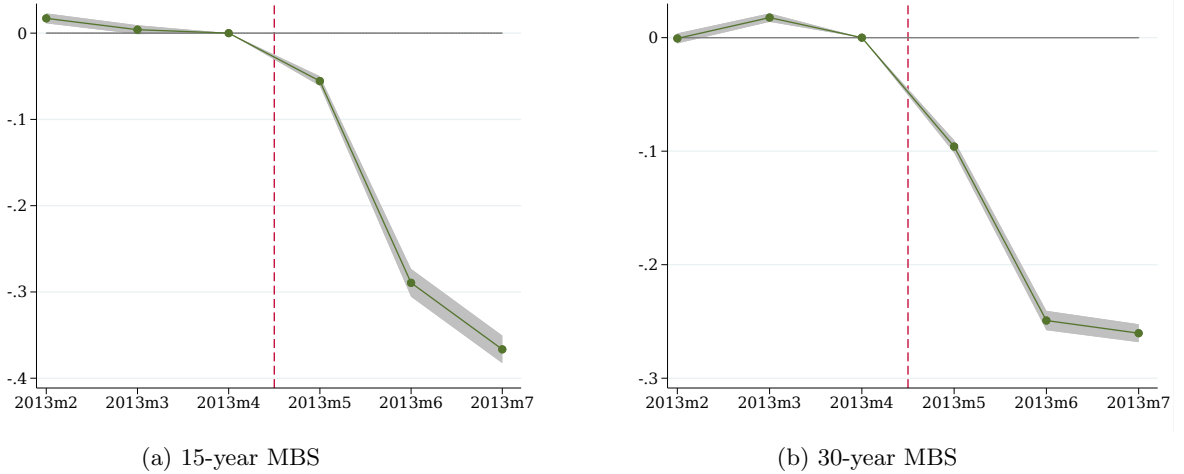
One concern about the TT event test is that our estimate might reflect a time-series trend on the degree of separation instead of TT itself. To address this concern, we estimate how the degree of separation changes in each month around the TT event. Precisely, we estimate Equation (6) with  $X_i(\beta)$  defined as follows:

$$X_i(\beta) = \sum_{t=2013:m2}^{2013:m7} 1[OriginationMonth_i = t]\beta_t, \quad (10)$$

where  $OriginationMonth_i$  refers to the loan origination month of loan  $i$ , and  $\beta_{2013:m4}$  is normalized to zero.

Figure 6 displays the estimates of  $\beta_t$  for 15- and 30-year MBS separately. In both panels, the degree of separation sharply decreases only for loans originated in May or later. This result suggests that the decrease in the degree of separation for such loans is likely due to the TT event. The drop in the degree of separation since May is gradual because of the following reasons. First, because the TT event took place in late May, lenders likely did not have enough time to respond to the event. Second, mortgage rates changed gradually between late May and late June, as shown earlier in Figure 4. Thus, loan value dispersion also changed gradually. Figure A.2 in Appendix B provides more detail on how the dispersion changed over time.

Figure 6: **Monthly Degree of Separation around Taper Tantrum:** This figure shows the coefficient estimates of  $\beta_t$  in Equation (10) and their 95% confidence intervals. Panels (a) and (b) report the estimates for 15- and 30-year MBS, respectively. Standard errors are clustered at the level of coupon $\times$ agency $\times$ state $\times$ lender $\times$ loan-size bin. Data source: eMBS.



### 6.3.3 Degree of Separation with Respect to LTV

Although loan size is the most important determinant of expected prepayments and loan value, there are other loan characteristics such as LTV that matters as well. Loans with very high LTV (typically greater than 80) tend to prepay slower because it typically takes a longer time for borrowers of such loans to build equity in their houses and have low enough LTVs to be able refinance to lower rates. In Appendix B.2, we provide lenders' pooling pattern with respect to LTV and provide estimates of the three tests using LTV as a proxy for loan values instead of loan size. We find that the results of the three tests are consistent with the model predictions.

### 6.3.4 Tests by Loan-Size Bins

Figure 1 shows that there is large variation in the degree of separation across loan sizes among low-balance loans and that the degree of separation changes discretely at certain cutoffs. One potential concern is that our main results could be driven by certain loan sizes. To rule out this possibility, we re-run the three tests but let  $\beta$ s be different for each loan-size bin. Figure A.6 in Appendix C graphically presents the coefficient estimates for each test by loan-size bins. It is apparent that similar test results hold for all loan-size bins. Similarly, these additional results show that our test results are robust.

## 7 MBS Pooling and Adverse Selection

Lenders' pooling decisions are important because, in theory, they impact adverse selection in the TBA market, which subsequently impacts TBA market liquidity and investors' trading costs. Proposition 2 shows that adverse selection is decreasing with the degree of pooling, or equivalently, that adverse selection in the TBA market is increasing with the degree of separation. Proposition 5 shows that if loan value dispersion increases, adverse selection becomes more severe, which is amplified by a concurrent increase in the degree of separation. In this section, we provide empirical evidence that the degree of pooling affects the TBA market adverse selection.

### 7.1 Empirical Predictions

We provide two sets of evidence that is consistent with a higher degree of separation increasing adverse selection in the TBA market.

**Ginnie Mae II Multi-Issuer MBS** In the first test, we compare Ginnie Mae II multiple issuer securities with Fannie Mae and Freddie Mac securities. Under the Ginnie Mae II multi-issuer program, Ginnie Mae issues exactly one MBS per month per coupon, and all lenders contribute their loans to a single MBS.<sup>22</sup> Also, there is a separate TBA market for the Ginnie Mae II multi-issuer securities. Thus, Ginnie Mae II MBS can be thought of as the social planner solution in which the degree of pooling equals one. Although there is still exogenous heterogeneity across Ginnie Mae MBS that were issued in different months, there is no heterogeneity across MBS with the same coupon that are issued in the same month. Thus, the degree of adverse selection in the TBA market should be lower for Ginnie TBAs than Fannie Mae or Freddie Mac TBAs. Because the difference in the degree of separation is rather exogenous in this test, a significant part of the resulting difference in adverse selection is likely due to the difference in the degree of separation.<sup>23</sup>

**Different MBS Coupons** In the second test, we compare degrees of adverse selection across different MBS coupons. As shown in Section 6, loan value dispersion is higher for loans in higher coupon Fannie Mae and Freddie Mac MBS, leading to a higher degree of separation in those loans. Thus, Proposition 5 predicts that the degree of adverse selection will also be higher for higher coupons. An important caveat is that higher loan value dispersion may lead to higher adverse selection even if the degree of separation remains the same, as shown in Proposition 3 and 5.<sup>24</sup> Thus, although empirical evidence for a higher degree of adverse selection for higher coupons is still consistent with the model, it should not necessarily be interpreted as causal evidence that the degree of separation impacts adverse selection.

## 7.2 Measuring Adverse Selection

We construct two different empirical measures of adverse selection. They capture relative differences in value of MBS traded between the TBA and SP markets. The first measure, *adv1*, uses the average original balance of loans in MBS  $m$  ( $avgbal_m$  as defined in Equation (2)) and is defined as the ratio of the weighted average  $avgbal$  of MBS delivered in the TBA market to the weighted average  $avgbal$  of MBS traded via SP. We calculate this ratio for each agency  $j$ , coupon  $k$ , and month  $t$  combinations for 30-year MBS. Assuming that

<sup>22</sup>For convenience, we will sometimes refer to the Ginnie Mae II multi-issuer MBS as Ginnie Mae MBS.

<sup>23</sup>There are numerous differences between loans in Ginnie Mae MBS and those in GSE MBS because Ginnie Mae MBS mostly comprises of loans issued through Federal Housing Administration (FHA) and U.S. Department of Veterans Affairs (VA). One possibility is that Ginnie Mae MBS have slower prepayments than GSE MBS, leading to lower MBS heterogeneity and ultimately lower adverse selection. However, given the stark pooling practice of Ginnie Mae II multi-issuer MBS program, we can, with relatively high confidence, say that the difference in MBS heterogeneity between Ginnie Mae and GSEs are mostly driven by the difference in pooling practices.

<sup>24</sup>Proposition 3 is fully subsumed in Proposition 5.

there are  $N_{jkt}$  cusips that can be delivered against agency  $j$ -coupon  $k$  pair on month  $t$ ,

$$adv1_{jkt} = \left( \frac{\sum_{m=1}^{N_{jkt}} w_{mt}^{tba} avgbal_m / \sum_{m=1}^{N_{jkt}} w_{mt}^{tba}}{\sum_{m=1}^{N_{jkt}} w_{mt}^{sp} avgbal_m / \sum_{m=1}^{N_{jkt}} w_{mt}^{sp}} - 1 \right) \times 100 \quad (11)$$

where  $w_{mt}^{tba}$  is the amount of MBS  $m$  that are delivered to the SOMA portfolio on month  $t$ , and  $w_{mt}^{sp}$  is the SP trading volume of MBS  $m$  on month  $t$  calculated from the regulatory TRACE data. Because we are using the SOMA portfolio data to proxy for TBA trading, we cannot calculate  $adv1$  for agency, coupon, and month combinations of which the Fed did not purchase enough MBS.

The higher the  $adv1$ , the more severe the adverse selection. Because MBS with lower  $avgbal$  tend to have lower prepayments and thus higher value,  $adv1$  is inversely related to the average value of MBS traded in TBA relative to that in SP.  $adv1$  of zero would imply that the average  $avgbal$  traded via TBA and SP markets are roughly the same, and  $adv1$  value of 10, for instance, would imply that the  $avgbal$  of MBS traded via TBA is 10% higher than those traded via SP.

The second adverse selection measure,  $adv2$ , uses the MBS price data. We define  $adv2$  as the ratio of the weighted average price of MBS traded in the SP market to the weighted average price of MBS delivered in the TBA market. Specifically,

$$adv2_{jkt} = \left( \frac{\sum_{m=1}^{N_{jkt}} w_{mt}^{sp} p_{mt} / \sum_{m=1}^{N_{jkt}} w_{mt}^{sp}}{\sum_{m=1}^{N_{jkt}} w_{mt}^{tba} p_{mt} / \sum_{m=1}^{N_{jkt}} w_{mt}^{tba}} - 1 \right) \times 10000 \quad (12)$$

where  $p_{mt}$  is the price of MBS  $m$  on month  $t$ , and  $w_{mt}^{tba}$ ,  $w_{mt}^{sp}$ , and  $N_{jkt}$  is defined as before. Higher values of  $adv2$  would also imply more severe adverse selection.  $adv2$  value of 10, for instance, would imply that the price of MBS traded via SP is 10 basis points (bps) higher than those traded via TBA. To get a sense of the magnitude, the unconditional average of the cross-sectional standard deviation of MBS price within the same agency-coupon is 19.1 bps.<sup>25</sup>

Neither of the two measures,  $adv1$  and  $adv2$ , is a perfect measure of adverse selection; rather, they are complementary. On the one hand,  $adv1$  does not capture the extent of adverse selection due to other determinants of loan value such as LTV and loan age because the loan size is only one of the determinants

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<sup>25</sup>This number is calculated in the following way. First, for each agency and coupon combination, we take the standard deviation of the standardized MBS prices. Standardized price for an MBS is calculated as the ratio between the MBS price and the average price of all MBS in the agency, coupon, and month combination. We then calculate the average of the standard deviation (across all agencies and coupons), and get 19.1 bps.

of loan value, albeit a major one. In contrast, *adv2* captures all of them since it uses prices. On the other hand, *adv2* may vary due to factors other than lenders' pooling and trading decisions. For example, a change in loan value dispersion may affect *adv2* through its impact on  $p_{mt}$  even if lenders' pooling and trading strategies are fixed. In contrast, *adv1* is not subject to this issue because it directly depends on lenders' pooling and trading strategies. For these reasons, we consider both adverse selection measures. To calculate the adverse selection measures, we use all MBS regardless of whether they were pooled and sold directly by lenders or whether they were pooled by the agencies because TBA sellers can deliver either of those MBS.

### 7.3 Summary Statistics and Test Results

**Summary Statistics** Table 2 presents the summary statistics of *adv1*, *adv2*, and other relevant outcomes by agency and coupon. It is worth noting that the degree of adverse selection is sizable for some agency and coupon combinations. For instance, for loans in 4% Fannie Mae MBS, *adv1* is 25.9, meaning that *avgbal* of MBS delivered via TBA is 25.9% higher than those delivered by SP. The value of *adv2* is 26.8 for the same sample, implying that the average price of MBS traded through SP is 26.8 bps higher than those delivered through TBA.

It is apparent that adverse selection varies across agencies and coupons in a way that is consistent with our predictions. First, both *adv1* and *adv2* are lower for Ginnie Mae MBS than for Fannie Mae or Freddie Mac MBS. In fact, *adv1* is close to zero and *adv2* is quite low for Ginnie Mae, implying that the value of the MBS delivered through the TBA and SP markets are not too different. *adv1* and *adv2* are different across the agencies mainly because the degree of separation is zero for Ginnie Mae due to its pooling practice. As in Section 6, the degree of separation is measured as the share of low-balance loans (those with size up to \$175,000) in low-balance MBS (MBS with maximum loan size up to \$175,000). The zero degree of separation mechanically results in no low-balance MBS for Ginnie Mae, although shares of low-balance loans in newly originated loans are comparable across the agencies.

Lower adverse selection in Ginnie Mae could be due to its lower loan value dispersion. As shown in Section 6.1, refinance incentives and loan value dispersion are positively correlated. In fact, loans in Ginnie Mae MBS have lower refinancing incentives, measured as the average difference between the weighted average coupon (WAC) of loans in MBS and the prevailing interest rate.<sup>26</sup> However, despite the relatively large variation in refinancing incentives across coupons within Ginnie Mae MBS, *adv1* stays almost constant, and

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<sup>26</sup>We use the 30-year rate from the Freddie Mac Primary Mortgage Market Survey for the prevailing interest rate.

$adv2$  varies relatively little. Thus, differences in loan value dispersion across the agencies are unlikely to explain lower adverse selection for Ginnie Mae.

**Table 2: Adverse Selection Measure:** In the table below, we present the average of  $adv1$ ,  $adv2$ , refinance incentive, the share of low-balance MBS, the share of low-balance loans, and the degree of separation by agency and coupon. We first calculate these values at the agency-coupon-month level and take the average across months. Refinance incentives, the share of low-balance MBS, the share of low-balance loans, and the degree of separation are calculated using MBS and loans in the MBS issued in the given month only. Refinance incentive is calculated as the weighted average of the difference between original WAC (weighted average coupon at issuance) of loans in MBS and the prevailing mortgage rate, using the issue amount as the weight. The share of low-balance MBS is weighted by the issue amount of the MBS. The share of low-balance loans and the degree of separation are weighted by the loan size. The share of low-balance loans and the degree of separation for Ginnie Mae uses data from October 2013 through December 2015 because earlier data are not available, and the rest of the table uses data from 2012 through 2015.

	Freddie Mac			Fannie Mae			Ginnie Mae		
	3%	3.5%	4%	3%	3.5%	4%	3%	3.5%	4%
$adv1$	7.788	14.758	26.882	9.683	17.355	25.889	0.155	0.425	-0.981
$adv2$	2.516	13.292	30.506	4.105	12.78	26.754	0.532	0.142	6.412
refinance incentive (refi)	-0.101	0.188	0.546	-0.094	0.2	0.532	-0.365	-0.049	0.303
share of low-bal MBS	0.061	0.147	0.238	0.069	0.161	0.261	0	0	0
share of low-bal loan	0.136	0.219	0.312	0.122	0.203	0.303	0.214	0.283	0.361
degree of separation	0.476	0.694	0.804	0.424	0.667	0.821	0	0	0

Second, both  $adv1$  and  $adv2$  are greater for higher coupons for Fannie Mae and Freddie Mac MBS. Other outcomes in Table 2 vary with coupons in a way that is consistent with model predictions. In Section 6, we showed that higher-coupon MBS have higher degrees of separation because loans in higher-coupon MBS have higher value dispersion due to larger refinancing incentives. These patterns are also shown in Table 2. Moreover, higher-coupon MBS have greater shares of low-balance loans, so these two patterns together lead to relatively more low-balance MBS in higher coupons.

However, it is not clear how much of variation in  $adv1$  and  $adv2$  across coupons is due to the difference in the degree of separation or loan value dispersion because differences in loan value dispersion and share of low-value loans would also affect the degree of adverse selection even if the degree of separation is the same. Thus, this result across coupons should be taken as empirical evidence that is consistent with the predictions of our theoretical model but not necessarily as causal evidence that the degree of separation changes TBA market adverse selection. Nevertheless, comparing with Ginnie Mae MBS here also gives some evidence that pooling practices matter for adverse selection, which is consistent with Proposition 3. As noted before,  $adv1$  is almost constant across coupons for Ginnie Mae due to its pooling practice, although loan value is likely more dispersed for loans in higher-coupon MBS due to their refinancing incentives. Moreover, the relation

between *adv2* and coupons for Ginnie Mae is not monotonic.

**Regression Test** We now test the two predictions in regression settings. To compare Ginnie Mae MBS with Fannie Mae and Freddie Mac MBS, we run the following regression:

$$adv_{jkt} = \alpha + \beta 1[j = \text{Ginnie Mae}] + \delta refi_{jkt} + \xi_k + \xi_t + \epsilon_{jkt}, \quad (13)$$

where  $adv_{jkt}$  is the adverse selection measure for agency  $j$ , coupon  $k$ , and month  $t$ .  $refi_{jkt}$  is the measure of refinancing incentives shown in Table 2.  $\xi_k$  and  $\xi_t$  are coupon fixed effects and year-month fixed effects, respectively.

Results are reported in Table 3, and they are consistent with the summary statistics shown in Table 2. We find that *adv1* is about 21 lower for Ginnie Mae compared to Fannie Mae and Freddie Mac, and *adv2* is lower by 8. These estimates are large in magnitude given that the unconditional standard deviations of *adv1* and *adv2* are 12.16 and 16.36, respectively. These large estimates suggest that the difference in pooling practices across the agencies is an important determinant of the degree of adverse selection in the TBA market.

**Table 3: Difference in Adverse Selection Across Agencies:** The following table presents the coefficient estimates from regression (13). We estimate the regression separately with the two measures of adverse selection. The estimation sample includes MBS for all of the three agencies. We use heteroskedasticity-robust standard errors. The stars indicate: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01. Data source: eMBS, TRACE, and ICE Data Pricing & Reference Data, LLC

	adv1			adv2		
	(1)	(2)	(3)	(4)	(5)	(6)
1[Agency=Ginnie]	-17.508*** (0.776)	-15.367*** (0.846)	-21.269*** (2.704)	-13.331*** (1.372)	-6.674*** (1.196)	-7.521** (3.125)
refi		8.521*** (1.394)	-15.248 (9.964)		26.500*** (2.461)	22.062* (12.166)
Constant	17.439*** (0.714)	15.499*** (0.698)	13.719*** (3.303)	15.465*** (1.180)	9.431*** (0.751)	0.300 (2.813)
yr-mon f.e.	No	No	Yes	No	No	Yes
coupon f.e.	No	No	Yes	No	No	Yes
Observations	331	331	331	331	331	331
Adjusted R <sup>2</sup>	0.451	0.519	0.611	0.142	0.509	0.591

For the second test, we run the following regression to study how adverse selection varies with coupons:

$$adv_{jkt} = \alpha + \beta_1 1[k = 3.5\%] + \beta_2 1[k = 4\%] + \xi_t + \epsilon_{jkt}. \quad (14)$$

We estimate regressions separately using Fannie Mae and Freddie Mac data and Ginnie Mae data. Results indicate that in Fannie Mae and Freddie Mac securities, higher coupons have higher *adv1* and *adv2*, and the magnitudes are economically large. On the other hand, *adv1* values do not vary across coupons for Ginnie Mae securities, and *adv2* varies to a lesser degree across coupons. The difference in results for Ginnie Mae between the two measures is because *adv2* captures the adverse selection due to heterogeneity in MBS values across MBS of different ages.

Table 4: **Difference in Adverse Selection Across Coupons:** The following table presents the coefficient estimates from regression (14). We estimate the regression separately with the two measures of adverse selection. Columns (1), (2), (5), and (6) report estimates for Fannie Mae and Freddie Mac MBS. The rest of the columns report estimates for Ginnie Mae MBS. We use heteroskedasticity-robust standard errors. The stars indicate: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01. Data source: eMBS, TRACE, and ICE Data Pricing & Reference Data, LLC

	adv1				adv2			
	Fannie & Freddie		Ginnie		Fannie & Freddie		Ginnie	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1[coupon=3.5%]	7.288*** (1.207)	8.988*** (1.251)	0.270 (0.547)	0.186 (0.898)	9.698*** (1.495)	18.520*** (1.754)	-0.390 (0.274)	2.319* (1.404)
1[coupon=4%]	17.625*** (1.536)	20.895*** (1.710)	-1.136 (0.939)	-0.895 (1.403)	25.318*** (2.497)	38.276*** (2.306)	5.881*** (2.153)	9.320*** (2.630)
Constant	8.768*** (0.943)	0.966 (4.153)	0.155 (0.468)	-1.330 (1.088)	3.338*** (0.503)	-18.212*** (2.562)	0.532** (0.248)	-6.132 (4.127)
yr-mon f.e.	No	Yes	No	Yes	No	Yes	No	Yes
Observations	225	225	106	106	225	225	106	106
Adjusted R <sup>2</sup>	0.394	0.475	0.020	-0.110	0.300	0.674	0.138	0.088

Given our results on the degree of separation for Fannie Mae and Freddie Mac in Section 6, one may ask whether the adverse selection is different in these two markets. However, a sizable chunk of Fannie Mae and Freddie Mac MBS are multi-lender pools, for which the agencies can decide on which loans to pool together. Because the agencies likely have incentives to internalize the effects of their pooling decisions on the performance of MBS trading, their pooling decisions can be markedly different from private lenders. In fact, Freddie Mac tends to create fewer low-balance MBS than Fannie Mae, which is opposite to private lenders' pooling behaviors reported in Section 6. As a result, the summary statistics in Table 2 indicate

that the overall degree of separation that includes both MBS swaps and multi-lender pools is very similar between Fannie Mae and Freddie Mac.

Similarly, one may wonder whether adverse selection decreases around the taper tantrum event, as our model would predict. Unfortunately, this prediction is difficult to test empirically. We can only measure adverse selection monthly for each agency and coupon pair, and the Fed mostly buys MBS with the currently prevailing coupon rate, which changes around the taper tantrum event. Therefore, there are very few data points, making statistical inference infeasible.

## 8 Conclusion

In this paper, we have shown theoretically and empirically that the cheapest-to-deliver incentives of the TBA market induce lenders to pool mortgages in a way that increases MBS heterogeneity and adverse selection problem in the TBA market. Our view is not that the TBA market structure is undesirable or makes the MBS market illiquid. In reality, the TBA market has long been liquid, which indicates that actual MBS heterogeneity has not been large enough to completely unravel the TBA market and that the liquidity benefit of the TBA market outweighs the cost from adverse selection. However, there is no guarantee that the market will remain liquid in the future because potential changes in policies—for example, a housing finance reform—may have large impacts on MBS heterogeneity and TBA market quality. Thus, it is important to take into account lenders’ pooling incentives when we evaluate policy proposals regarding the mortgage market.

In fact, the Federal Housing Finance Agency (FHFA), which regulates Fannie Mae and Freddie Mac, recently proposed changes to pooling practices for agency MBS to ensure relative homogeneity across securities. One of the issues to which FHFA paid attention is whether to limit the issuance amount of MBS consisting of high-value loans that are intended to trade in the SP market. FHFA is concerned that SP trading can potentially undermine the TBA market performance.<sup>27</sup> Our results show that this concern is indeed valid.

We also show that an increase in SP trading costs, holding TBA trading costs fixed, *increases* lender revenue. This result implies that decreasing SP trading costs—for instance, through SP market transparency—has an adverse impact on not only the TBA market but also the overall agency MBS market, which in turn may negatively affect mortgage borrowers.

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<sup>27</sup>See the document in the link for more information: [https://www.fhfa.gov/Media/PublicAffairs/PublicAffairsDocuments/Pooling\\_RFI.pdf](https://www.fhfa.gov/Media/PublicAffairs/PublicAffairsDocuments/Pooling_RFI.pdf)

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# A Proofs

In general, to solve the model, we will first show that it is sufficient to solve the model for  $a = 1$ . We will then prove the theorems and propositions for  $a \neq 1$ .

## A.1 Proofs for second stage equilibrium

We first show that to find the second stage equilibrium, it is sufficient to find the second stage equilibrium for  $a = 1$ .

**Lemma 1.** *Consider an alternative model in which  $x$  remains the same, but the trading cost parameters are  $\frac{c_0}{a}$ ,  $\frac{c_1}{a}$ , and  $\frac{\bar{c}}{a}$  and loan dispersion is 1 (that is, loans are distributed  $[0, 1]$ ). If  $(p, q)$  is the second stage equilibrium for the original model, then TBA price of  $\tilde{p} = \frac{p}{a}$  and TBA volume of  $\tilde{q} = q$  is a second stage equilibrium for this alternative model. Conversely, if  $(\tilde{p}, \tilde{q})$  is a second stage equilibrium for the alternative model, then  $(a\tilde{p}, \tilde{q})$  is a second stage equilibrium for the original model where the loan dispersion is  $a$ .*

*Proof.*  $p$  and  $q$  satisfy (3) and (4).  $\tilde{p}$  and  $\tilde{q}$  satisfy:

$$\tilde{p} = \frac{\frac{x^2}{2} \left( 1 - \tilde{F} \left( \frac{x}{2} - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) + \int_x^1 v \left( 1 - \tilde{F} \left( v - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) dv}{x \left( 1 - \tilde{F} \left( \frac{x}{2} - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) + \int_x^1 \left( 1 - \tilde{F} \left( v - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) dv}, \quad (15)$$

$$\tilde{q} = x \left( 1 - \tilde{F} \left( \frac{x}{2} - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) + \int_x^1 \left( 1 - \tilde{F} \left( v - \frac{c_0}{a} - \frac{c_1}{a} \tilde{q} - \tilde{p} \right) \right) dv \quad (16)$$

where  $\tilde{f}(\eta)$  and  $\tilde{F}(\eta)$  are PDF and CDF for  $\eta$  distributed uniformly on  $[0, \frac{\bar{c}}{a}]$ . It is straightforward to show that if  $p$  and  $q$  satisfy (3) and (4),  $\tilde{p} = \frac{p}{a}$  and  $\tilde{q} = q$  satisfy (15) and (16). It is also straightforward to show that if  $\tilde{p}$  and  $\tilde{q}$  satisfy (15) and (16),  $p = a\tilde{p}$  and  $q = \tilde{q}$  satisfy (3) and (4).  $\square$

**Proof of Proposition 1:**

From (3), we get  $\frac{ax}{2} \leq p(x) < a$ . Hence, the lowest value pool will always be sold in the TBA market.  $\square$

From Lemma 1, we can solve for the second stage equilibrium easily if we know the solution for  $a = 1$ . We first prove the results for  $a = 1$ .

Before we show Theorem 1, we first show a few Lemmas that will be used in the proofs. Given Proposition 1, assuming  $a = 1$ , we can rewrite (3) and (4) as

$$p = \frac{\frac{x^2}{2} + \int_x^1 v (1 - F(v - c_0 - c_1 q - p)) dv}{x + \int_x^1 (1 - F(v - c_0 - c_1 q - p)) dv} \quad (17)$$

$$q = x + \int_x^1 (1 - F(v - c_0 - c_1 q - p)) dv \quad (18)$$

Define

$$H(p, q; x) := \frac{x^2}{2} - px + \int_x^1 (v - p) (1 - F(v - c_0 - c_1 q - p)) dv \quad (19)$$

$$= \frac{1}{2} - p - \int_x^1 (v - p) F(v - c_0 - c_1 q - p) dv \quad (20)$$

$$G(p, q; x) := 1 - q - \int_x^1 F(v - c_0 - c_1 q - p) dv \quad (21)$$

Finding  $p$  and  $q$  that satisfy (17) and (18) is equivalent to finding  $p$  and  $q$  that satisfy  $H(p, q; x) = 0$  and  $G(p, q; x) = 0$ .

Define  $p(q; x)$  as the  $p$  that satisfies  $H(p, q; x) = 0$  given  $q$  and  $x$ . Similarly, define  $q(p; x)$  as the  $q$  that satisfies  $G(p, q; x) = 0$  given  $p$  and  $x$ .

**Lemma 2.** If  $a = 1$ ,

$$q(x) \geq x, \quad q(x) \geq c_0 + p(x).$$

*Proof.* From (18), it is pretty obvious that  $q(x) \geq x$ .

If  $x \geq c_0 + p(x)$ , then  $q(x) \geq x \geq c_0 + p(x)$ . If  $x < c_0 + p(x)$ ,

$$q(x) = x + \int_x^1 (1 - F(v - c_0 - c_1 q - p)) dv \geq x + \int_x^{c_0 + p(x)} (1 - F(v - c_0 - c_1 q - p)) dv = c_0 + p(x).$$

□

**Lemma 3.** If  $a = 1$ , for a given  $q \in [x, 1]$  and  $x \in [0, 1]$ , a unique  $p(q, x) \in (\frac{x}{2}, 1)$  exists such that  $H(p, q; x) = 0$ .  $p(q, x)$  is a continuous function in  $q$ .

*Proof.*

$$\begin{aligned} H(p = \frac{x}{2}, q; x) &= \frac{1}{2} - \frac{x}{2} - \int_x^1 \left(v - \frac{x}{2}\right) F\left(v - c_0 - c_1 q - \frac{x}{2}\right) dv \\ &> \frac{1}{2} - \frac{x}{2} - \int_x^1 \left(v - \frac{x}{2}\right) dv = 0 \\ H(p = 1, q; x) &= \frac{1}{2} - 1 - \int_x^1 (v - 1) F(v - c_0 - c_1 q - 1) dv < 0 \end{aligned}$$

Since  $H$  is continuous in  $p$ , there exists a  $p(q, x) \in (\frac{x}{2}, 1)$  such that  $H(p, q; x) = 0$ .

$$\begin{aligned} \frac{\partial H(p, q; x)}{\partial p} &= -1 + \int_x^1 F(v - c_0 - c_1 q - p) + (v - p) f(v - c_0 - c_1 q - p) dv \\ &= \begin{cases} -1 + \frac{1}{e}(1 - x)(1 + x - c_0 - c_1 q - 2p) < -x^2 - (c_0 + c_1 q + 2p)(1 - x) \leq 0 & \text{if } x \geq c_0 + c_1 + p \\ -1 + \frac{1}{e}(1 - c_0 - c_1 q - p)(1 - p) < -(c_0 + c_1 q + p)(1 - p) - p < 0 & \text{otherwise} \end{cases} \end{aligned}$$

Since  $\frac{\partial H(p, q; x)}{\partial p} < 0$ , such  $p(q, x)$  is unique. Using the implicit function theorem, since  $\frac{\partial H(p, q; x)}{\partial p} \neq 0$ ,  $p(q, x)$  is a continuous function in  $q$ . □

**Lemma 4.** If  $a = 1$ , for a given  $p \in [\frac{x}{2}, 1]$  and  $x \in [0, 1]$ , a unique  $q(p, x) \in [x, 1]$  exists such that  $G(p, q; x) = 0$ .  $q(p, x)$  is a continuous function in  $p$ .

*Proof.*

$$\begin{aligned} G(p, q = x; x) &= 1 - x - \int_x^1 F(v - c_0 - c_1 x - p) dv = (1 - x) - (1 - x) = 0 \\ G(p, q = 1; x) &= 1 - 1 - \int_x^1 F(v - c_0 - c_1 - p) dv < 0 \end{aligned}$$

Since  $G$  is continuous in  $q$ , there exists a  $q(p, x) \in [x, 1]$  such that  $G(p, q; x) = 0$ .

$$\frac{\partial G(p, q; x)}{\partial q} = -1 + c_1 \int_x^1 f(v - c_0 - c_1 q - p) dv < -1 + c_1(1 - x) < 0$$

Thus, such  $q(p; x)$  is unique. Using the implicit function theorem, since  $\frac{\partial G(p, q; x)}{\partial q} \neq 0$ ,  $q(p, x)$  is a continuous function in  $p$ .  $\square$

**Proof of Theorem 1:** We first show the theorem under  $a = 1$ . Since unique  $p(q, x)$  and  $q(p, x)$  exist and are continuous in  $q$  and  $p$ , respectively,  $p(q(p; x); x) : [\frac{x}{2}, 1] \rightarrow [\frac{x}{2}, 1]$  is a continuous function in  $p$ . By the fixed point theorem, there exists a fixed point  $p$  that satisfies  $p = p(q(p; x))$ . Define such  $p$  as  $p(x)$ , and define  $q(x) = q(p(x); x)$ .

We now need to show that the solution  $p(x)$ ,  $q(x)$  is unique. To do so, it is sufficient to show that  $\frac{dp(q(p; x); x)}{dp} < 1$ .

$$\frac{dp(q(p; x); x)}{dp} = \frac{\partial p(q(p; x); x)}{\partial q} \frac{\partial q(p; x)}{\partial p}$$

We will show that  $\frac{\partial p(q; x)}{\partial q} < 1$  and  $\frac{\partial q(p; x)}{\partial p} < 1$ .

We look at  $\frac{\partial p(q; x)}{\partial q}$  first. Since  $p(q; x)$  is the  $p$  that satisfies  $H(p; q, x) = 0$ , we get

$$H(p; q, x) = \frac{1}{2} - p - \int_x^1 (v - p) F(v - c_0 - c_1 q - p) dv = 0 \quad (22)$$

From the Implicit Function Theorem,

$$\begin{aligned} \frac{\partial p(q; x)}{\partial q} &= - \frac{\frac{\partial H}{\partial q}}{\frac{\partial H}{\partial p}} \\ &= \frac{c_1 \int_x^1 (v - p) f(v - c_0 - c_1 q - p) dv}{1 - \int_x^1 [F(v - c_0 - c_1 q - p) + (v - p) f(v - c_0 - c_1 q - p)] dv} \end{aligned}$$

Thus, we want to show

$$\int_x^1 [F(v - c_0 - c_1 q - p) + (c_1 + 1)(v - p) f(v - c_0 - c_1 q - p)] dv < 1 \quad (23)$$

Denoting  $z = \max(x, c_0 + c_1 q + p)$ , if  $z > 1$ , (23) holds. If  $z \leq 1$ ,

$$\begin{aligned} &\int_x^1 [F(v - c_0 - c_1 q - p) + (c_1 + 1)(v - p) f(v - c_0 - c_1 q - p)] dv \\ &= \int_z^1 [F(v - c_0 - c_1 q - p) + (c_1 + 1)(v - p) f(v - c_0 - c_1 q - p)] dv \\ &= \frac{1}{\bar{c}} \left[ \frac{c_1 + 2}{2} (1 - z^2) - (c_0 + c_1 q + (c_1 + 2)p)(1 - z) \right] \\ &< \frac{1}{\bar{c}} \frac{c_1 + 2}{2} < 1 \quad (\because c_1 + 1 < \bar{c}) \end{aligned}$$

So we have shown that  $\frac{\partial p(q; x)}{\partial q} < 1$ .

We now show that  $\frac{\partial q(p; x)}{\partial p} < 1$ .

$$\frac{\partial q(p; x)}{\partial p} = - \frac{\frac{\partial G}{\partial p}}{\frac{\partial G}{\partial q}} = \frac{\int_x^1 f(v - c_0 - c_1 q - p) dv}{1 - c_1 \int_x^1 f(v - c_0 - c_1 q - p) dv}$$

(i) If  $1 \geq x \geq c_0 + c_1 q + p$ :

$$\frac{\partial q(p; x)}{\partial p} = \frac{1 - x}{\bar{c} - c_1(1 - x)}$$

$$\frac{\partial q(p; x)}{\partial p} < 1 \Leftrightarrow (c_1 + 1)(1 - x) < \bar{c}$$

Last inequality holds since  $c_1 + 1 < \bar{c}$ .

(ii)  $1 \geq c_0 + c_1 q + p > x$ :

Proof is essentially same. Last inequality would be  $(c_1 + 1)(1 - c_0 - c_1 q - p) < \bar{c}$ .

(iii)  $c_0 + c_1 q + p > x$  and  $c_0 + c_1 q + p \geq 1$ :

$$\frac{\partial q(p; x)}{\partial p} = 0.$$

Thus,  $\frac{\partial q(p; x)}{\partial p} < 1$ .

Putting them together, we get  $\frac{dp(q(p; x); x)}{dp} < 1$ . Thus, the fixed point in which  $p = p(q(p; x); x)$  is unique. Thus, for a given  $x$ ,  $p(x)$  and  $q(x)$  pair is unique.

We now consider cases in which  $a \neq 1$ . Denote  $p$  and  $q$  under parameters  $c$  and  $a$  as  $p(x; c, a)$  and  $q(x; c, a)$ . Since  $p(x; \frac{c}{a}, 1)$  and  $q(x; \frac{c}{a}, 1)$  exist and are unique,  $p(x; c, a) = ap(x; \frac{c}{a}, 1)$  and  $q(x; c, a) = q(x; \frac{c}{a}, 1)$  exist and are unique.  $\square$

Before we prove Proposition 2, we show the following lemmas.

**Lemma 5.** *If  $a = 1$ ,  $p(x)$  and  $q(x)$  are weakly increasing in  $x$ .*

*Proof.* Using the fixed point argument used in the proof of Theorem 1, to show that  $p(x)$  is weakly increasing in  $x$ , it is sufficient to show that for a fixed  $p$ ,  $p(q(p; x); x)$  is weakly increasing in  $x$ . This implies that the  $p(q(p; x); x)$  graph shifts right when  $x$  increases, thus the fixed point  $p(x)$  also increases.

$$\begin{aligned} \frac{dp(q(p; x); x)}{dx} &= \frac{\partial p(q(p; x); x)}{\partial x} + \frac{\partial p(q(p; x); x)}{\partial q} \frac{\partial q(p; x)}{\partial x} \\ \frac{\partial p(q(p; x); x)}{\partial x} &= -\frac{\frac{\partial H}{\partial x}}{\frac{\partial H}{\partial p}} \\ \frac{\partial p(q(p; x); x)}{\partial q} &= -\frac{\frac{\partial H}{\partial q}}{\frac{\partial H}{\partial p}} \\ \frac{\partial q(p; x)}{\partial x} &= -\frac{\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial q}} \\ \frac{\partial H}{\partial x} &= (x - p)F(x - c_0 - c_1 q - p) \geq 0 \\ \frac{\partial H}{\partial p} &= -1 + \int_x^1 [F(v - c_0 - c_1 q - p) + (v - p)f(v - c_0 - c_1 q - p)]dv \\ &= -1 + \frac{1}{\bar{c}} [v^2 - (c_0 + c_1 q + p)v]_{\max(x, c_0 + c_1 q + p)}^1 \leq -1 + \frac{1}{\bar{c}} [v^2]_{\max(x, c_0 + c_1 q + p)}^1 < -1 + (1 - x^2) \leq 0 \\ \frac{\partial G}{\partial x} &= F(x - c_0 - c_1 q - p) \geq 0 \\ \frac{\partial G}{\partial q} &= -1 + c_1 \int_x^1 f(v - c_0 - c_1 q - p)dv < 0 \end{aligned} \tag{24}$$

Putting everything together into (24), we get that  $p(q(p; x); x)$  is weakly increasing in  $x$ . Thus,  $p(x)$  is weakly increasing in  $x$ . Since  $q(x) = q(p(x); x)$  and  $\frac{q(p; x)}{\partial x} \geq 0$  (shown above), it is sufficient to show that  $\frac{q(p; x)}{\partial p} \geq 0$ .

$$\begin{aligned}\frac{\partial G}{\partial p} &= \int_x^1 f(v - c_0 - c_1 q - p) dv \geq 0 \\ \frac{q(p; x)}{\partial p} &= -\frac{\frac{\partial G}{\partial p}}{\frac{\partial G}{\partial q}} \geq 0\end{aligned}$$

Thus,  $p(x)$  and  $q(x)$  are weakly increasing in  $x$ . □

**Lemma 6.** Assume  $a = 1$ . Define  $p_0 = p(x = 0)$  and  $q_0 = q(x = 0)$ . Then the following are true:

1. If  $x \leq c_0 + c_1 q_0 + p_0$ , then  $p(x) = p_0$  and  $q(x) = q_0$ .
2. If  $x > c_0 + c_1 q_0 + p_0$ , then  $x > c_0 + c_1 q(x) + p(x)$ .

*Proof.* We will show that if  $x \leq c_0 + c_1 q_0 + p_0$ , then  $p(x) = p_0$  and  $q(x) = q_0$ . Since  $p(x)$  and  $q(x)$  are weakly increasing in  $x$  from Lemma 5, this is sufficient to show the proposition.

$(p_0, q_0)$  is the solution to  $H(p, q; x = 0) = 0$  and  $G(p, q; x = 0) = 0$ .

$$\begin{aligned}0 = H(p, q; 0) &= \frac{1}{2} - p + \int_0^1 (v - p)(1 - F(v - c_0 - c_1 q - p)) dv \\ &= \frac{1}{2} - p + \int_{c_0 + c_1 q + p}^1 (v - p)(1 - F(v - c_0 - c_1 q - p)) dv\end{aligned}\tag{25}$$

$$\begin{aligned}0 = G(p, q; 0) &= 1 - q - \int_0^1 F(v - c_0 - c_1 q - p) dv \\ &= 1 - q - \int_{c_0 + c_1 q + p}^1 F(v - c_0 - c_1 q - p) dv\end{aligned}\tag{26}$$

Assume  $x \leq c_0 + c_1 q(x) + p(x)$ . Then  $(p(x), q(x))$  is the solution to the following system of equations:

$$\begin{aligned}0 = H(p, q; x) &= \frac{1}{2} - p + \int_x^1 (v - p)(1 - F(v - c_0 - c_1 q - p)) dv \\ &= \frac{1}{2} - p + \int_{c_0 + c_1 q + p}^1 (v - p)(1 - F(v - c_0 - c_1 q - p)) dv\end{aligned}\tag{27}$$

$$\begin{aligned}0 = G(p, q; x) &= 1 - q - \int_x^1 F(v - c_0 - c_1 q - p) dv \\ &= 1 - q - \int_{c_0 + c_1 q + p}^1 F(v - c_0 - c_1 q - p) dv\end{aligned}\tag{28}$$

(25) is the same as (27), and (26) is same as (28). Since we know that the solution is unique from Theorem 1,  $p(x) = p_0$  and  $q(x) = q_0$ . □

**Proof of Proposition 2:** We first show the proposition for  $a = 1$ . We already know that  $p(x)$  and  $q(x)$  are weakly increasing in  $x$  from Lemma 5. Hence, it is sufficient to show that  $p(x)$  and  $q(x)$  are strictly increasing in  $x$  if  $x > c_0 + c_1 q_0 + p_0$ .

$(p(x), q(x))$  is the solution to the following system of equations:  $H(p, q; x) = 0$  and  $G(p, q; x) = 0$ . By the Implicit function theorem,

$$\begin{pmatrix} \frac{\partial p(x)}{\partial x} \\ \frac{\partial q(x)}{\partial x} \end{pmatrix} = - \begin{pmatrix} \frac{\partial H(p, q; x)}{\partial p} & \frac{\partial H(p, q; x)}{\partial q} \\ \frac{\partial G(p, q; x)}{\partial p} & \frac{\partial G(p, q; x)}{\partial q} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial H(p, q; x)}{\partial x} \\ \frac{\partial G(p, q; x)}{\partial x} \end{pmatrix}.\tag{29}$$

Define

$$A(p, q; x) = \begin{pmatrix} \frac{\partial H(p, q; x)}{\partial p} & \frac{\partial H(p, q; x)}{\partial q} \\ \frac{\partial G(p, q; x)}{\partial p} & \frac{\partial G(p, q; x)}{\partial q} \end{pmatrix}. \quad (30)$$

For  $x > c_0 + c_1 q_0 + p_0$ ,

$$\begin{aligned} \frac{\partial H}{\partial p} &= -1 + \int_x^1 [F(v - c_0 - c_1 q - p) + (v - p)f(v - c_0 - c_1 q - p)]dv \\ &= -1 + \frac{1}{\bar{c}}(1 - x^2 - (c_0 + c_1 q + 2p)(1 - x)) \\ &< -1 + (1 - x^2 - (c_0 + c_1 q + 2p)(1 - x)) \leq 0 \\ \frac{\partial H}{\partial q} &= c_1 \int_x^1 (v - p)f(v - c_0 - c_1 q - p)dv \\ &= \frac{c_1}{\bar{c}} \left( \frac{1}{2} - \frac{x^2}{2} - p + px \right) \geq 0 \\ \frac{\partial G}{\partial p} &= \int_x^1 f(v - c_0 - c_1 q - p)dv = \frac{1 - x}{\bar{c}} \geq 0 \\ \frac{\partial G}{\partial q} &= -1 + c_1 \int_x^1 f(v - c_0 - c_1 q - p)dv = -1 + \frac{c_1(1 - x)}{\bar{c}} < 0 \end{aligned}$$

Thus,

$$\begin{aligned} \det(A) &= \frac{\partial H}{\partial p} \frac{\partial G}{\partial q} - \frac{\partial H}{\partial q} \frac{\partial G}{\partial p} \\ &= 1 - \frac{1}{\bar{c}}(1 - x^2 - (c_0 + c_1 q + 2p)(1 - x)) + \frac{c_1(1 - x)}{\bar{c}} \left( -1 + \frac{1}{2\bar{c}} - \frac{x^2}{2\bar{c}} - \frac{(c_0 + c_1 q + p)(1 - x)}{\bar{c}} \right) \\ &= 1 - \frac{1 - x}{\bar{c}}(1 + x + c_1) + \frac{(c_0 + c_1 q + p)(1 - x)}{\bar{c}} \left( 1 - \frac{c_1(1 - x)}{\bar{c}} \right) + \frac{p(1 - x)}{\bar{c}} + \frac{c_1(1 - x)}{2\bar{c}^2}(1 - x^2) \\ &> x \left( 1 - \frac{1 - x}{\bar{c}} \right) + \frac{(c_0 + c_1 q + p)(1 - x)}{\bar{c}} \left( 1 - \frac{c_1(1 - x)}{\bar{c}} \right) + \frac{p(1 - x)}{\bar{c}} + \frac{c_1(1 - x)}{2\bar{c}^2}(1 - x^2) \quad (\because c_1 + 1 < \bar{c}) \\ &> 0 \end{aligned}$$

(29) now becomes

$$\begin{pmatrix} \frac{\partial p(x)}{\partial x} \\ \frac{\partial q(x)}{\partial x} \end{pmatrix} = -\frac{1}{\det(A)} \begin{pmatrix} \frac{\partial G(p, q; x)}{\partial q} & -\frac{\partial H(p, q; x)}{\partial q} \\ -\frac{\partial G(p, q; x)}{\partial p} & \frac{\partial H(p, q; x)}{\partial p} \end{pmatrix} \begin{pmatrix} \frac{\partial H(p, q; x)}{\partial x} \\ \frac{\partial G(p, q; x)}{\partial x} \end{pmatrix}. \quad (31)$$

Because  $x > c_0 + c_1 q(x) + p(x)$  if  $x > c_0 + c_1 q_0 + p_0$  ( $\because$  Lemma 6),

$$\begin{aligned} \frac{\partial H}{\partial x} &= (x - p)F(x - c_0 - c_1 q_0 - p) > 0 \\ \frac{\partial G}{\partial x} &= F(x - c_0 - c_1 q - p) > 0. \end{aligned}$$

Thus, we get that  $p(x)$  and  $q(x)$  are strictly increasing in  $x$  if  $x > c_0 + c_1 q_0 + p_0$ . Since  $s(x) = 1 - \frac{p(x)}{a}$ ,  $s(x)$  is weakly decreasing in  $x$  and strictly decreasing if  $x > c_0 + c_1 q_0 + p_0$ .

Cases in which  $a \neq 1$  can be shown easily using the  $a = 1$  results and Lemma 1.  $\square$

We show the following lemma, which will be used to prove Propositions 3 and 5.

**Lemma 7.** Assume  $a = 1$ . If  $x < 1$ ,  $p(x)$  and  $q(x)$  are strictly increasing in cost differential parameters  $c_0$ ,  $c_1$ , and  $\bar{c}$ . If  $x = 1$ ,  $p(x)$  and  $q(x)$  are constant.

*Proof.* If  $x = 1$ ,  $p(x) = \frac{1}{2}$  and  $q(x) = 1$ , so are constant.

If  $x < 1$ , for  $c = c_0, c_1$ , or  $\bar{c}$ ,

$$\begin{pmatrix} \frac{\partial p(x;c)}{\partial c} \\ \frac{\partial q(x;c)}{\partial c} \end{pmatrix} = -\frac{1}{\det(A)} \begin{pmatrix} \frac{\partial G(p,q;x)}{\partial q} & -\frac{\partial H(p,q;x)}{\partial q} \\ -\frac{\partial G(p,q;x)}{\partial p} & \frac{\partial H(p,q;x)}{\partial p} \end{pmatrix} \begin{pmatrix} \frac{\partial H(p,q;x)}{\partial c} \\ \frac{\partial G(p,q;x)}{\partial c} \end{pmatrix} \quad (32)$$

where  $A$  is defined in (30).

From the proof for Proposition 2, we know that  $\det(A) > 0$  if  $x > c_0 + c_1 q_0 + p_0$ . We show that  $\det(A) > 0$  for  $x \leq c_0 + c_1 q_0 + p_0$  as well. If  $x \leq c_0 + c_1 q_0 + p_0$ , then  $x \leq c_0 + c_1 q + p$  (Lemma 9). Writing  $z = c_0 + c_1 q + p$  as short hand,

$$\begin{aligned} \det(A) &= \frac{\partial H}{\partial p} \frac{\partial G}{\partial q} - \frac{\partial H}{\partial q} \frac{\partial G}{\partial p} \\ &= \left(-1 + \frac{1}{\bar{c}}(1 + zp - z - p)\right) \left(-1 + \frac{c_1(1-z)}{\bar{c}}\right) - \frac{1-z}{\bar{c}} \frac{c_1}{\bar{c}} \left(\frac{1}{2} - p - \frac{z^2}{2} + \frac{pz}{2}\right) \\ &= 1 - \frac{1}{\bar{c}}(1 + zp - z - p + c_1(1-z)) + \frac{1}{\bar{c}^2} c_1(1-z) \left(\frac{1}{2} + \frac{pz}{2}\right) \\ &= 1 - \frac{(c_1 + 1)(1-z)}{\bar{c}} + \frac{p(1-z)}{\bar{c}} + \frac{1}{\bar{c}^2} c_1(1-z) \left(\frac{1}{2} + \frac{pz}{2}\right) > 0 \quad (\because \bar{c} > c_1 + 1) \end{aligned}$$

Since  $c_0 + c_1 < \frac{1}{2}$ ,  $p \leq \frac{1}{2}$ , and  $x \leq q(x) \leq 1$ , for  $x < 1$ , we have:

$$\begin{aligned} \frac{\partial H(p, q, x; c)}{\partial c_0} &= \int_x^1 (v - p) f(v - c_0 - c_1 q - p) > 0 \\ \frac{\partial H(p, q, x; c)}{\partial c_1} &= q \int_x^1 (v - p) f(v - c_0 - c_1 q - p) > 0 \\ \frac{\partial H(p, q, x; c)}{\partial \bar{c}} &= \int_{\max(c_0 + c_1 q + p, x)}^1 (v - p)(v - c_0 - c_1 q - p(x)) \bar{c}^{-2} dv > 0 \end{aligned}$$

Thus,  $\frac{\partial p(x;c)}{\partial c} > 0$  and  $\frac{\partial q(x;c)}{\partial c} > 0$  for  $c_0, c_1, \bar{c}$ . □

**Proof of Proposition 3:** If  $x = 1$ ,  $p(x) = \frac{1}{2}a$  and  $q(x) = 1$ , so it is clear that  $q(x; c, a)$  and  $s(x; c, a) = 1 - \frac{p(x)}{a}$  are constants regardless of  $a$  or  $c$  values.

We now look at  $x < 1$ .

$$\begin{aligned} q(x; c, a) &= q(x; \frac{c}{a}, 1) \\ s(x; c, a) &= 1 - \frac{p(x; c, a)}{a} = 1 - p(x; \frac{c}{a}, 1) \end{aligned}$$

Thus, from Lemma 7, we know that  $q(s)$  is strictly increasing (decreasing) in  $c$  parameters and strictly decreasing (increasing) in  $a$ . □

## A.2 Proof of First Stage Problem

Similar to the proof for the second stage equilibrium, we first show that it is sufficient to solve for the first stage under  $a = 1$  and show how to transform the solution to cases in which  $a \neq 1$ .

**Lemma 8.** Consider an alternative model in which TBA price and quantity are  $\tilde{p}$  and  $\tilde{q}$ , and the trading cost parameters are  $\frac{c_0}{a}$ ,  $\frac{c_1}{a}$ , and  $\frac{\bar{c}}{a}$ , issuance cost function is  $\frac{K(x)}{a}$  and loan dispersion is 1 (that is, loans are distributed  $[0, 1]$ ). If lenders choose  $x(p, q)$  under the original model, then lenders choose  $\tilde{x}(\tilde{p}, \tilde{q}) = x(\frac{\tilde{p}}{a}, \tilde{q})$  under this alternative model. Conversely, if lenders

choose  $\tilde{x}(\tilde{p}, \tilde{q})$  under this alternative model, then lenders choose  $x(p, q) = \tilde{x}(ap, q)$  under the original model in which the loan dispersion is  $a$ .

*Proof.* We first show that the values of  $b_0$  and  $m$  does not matter for the first stage solution. Define

$$\begin{aligned}\bar{r}(v; p, q) &= r(v; p, q) + (b_0 - m(c_0 + c_1 q)) \\ \pi(x; p, q) &= x\bar{r}\left(\frac{ax}{2}; p, q\right) + \int_{ax}^a \frac{1}{a} \bar{r}(v; p, q) dv - K(x) - (b_0 - m(c_0 + c_1 q))\end{aligned}$$

Thus, the value of  $m$  and  $b_0$  does not matter. Therefore, in the alternative model, we will set  $m$  to be the same, and  $\tilde{b}_0 = \frac{b_0}{a}$ .

We then get

$$\begin{aligned}\bar{r}\left(\frac{v}{a}; \frac{p}{a}, q\right) &= \frac{p}{a} \left(1 - \bar{F}\left(\frac{v - c_0 - c_1 q - p}{a}\right)\right) + \frac{v - c_0 - c_1 q}{a} \bar{F}\left(\frac{v - c_0 - c_1 q - p}{a}\right) - \int_0^{\frac{1}{a}(v - c_0 - c_1 q - p)} \eta \tilde{f}(\eta) d\eta \\ &\quad - \frac{1}{a} (b_0 - m(c_0 + c_1 q)) = \frac{1}{a} r(v; p, q)\end{aligned}$$

$\tilde{f}$  and  $\tilde{F}$  are pdf and cdf for uniform distribution on  $[0, \frac{\bar{c}}{a}]$ . Thus,

$$\begin{aligned}\tilde{\pi}\left(x; \frac{p}{a}, q\right) &= \frac{1}{a} \pi(x; p, q). \\ \tilde{x}\left(\frac{p}{a}, q\right) &= x(p, q).\end{aligned}$$

□

If we denote the first stage solution as  $x(p, q; c, a, K(x))$ , then Lemma 8 implies that

$$x(p, q; c, a, K(x)) = x\left(\frac{p}{a}, q; \frac{c}{a}, 1, \frac{K(x)}{a}\right).$$

Before we move on, it is useful to calculate the first and second derivative of  $r(v)$ .

$$\begin{aligned}r(v; p, q) &= p(1 - F(v - c_0 - c_1 q - p)) + (v - c_0 - c_1 q)F(v - c_0 - c_1 q - p) - \int_0^{v - c_0 - c_1 q - p} \epsilon f(\epsilon) d\epsilon - (b_0 - m(c_0 + c_1 q)) \\ &= p + (v - c_0 - c_1 q - p)F(v - c_0 - c_1 q - p) - \int_0^{v - c_0 - c_1 q - p} \epsilon f(\epsilon) d\epsilon - (b_0 - m(c_0 + c_1 q)) \\ r'(v) &= F(v - c_0 - c_1 q - p) + (v - c_0 - c_1 q - p)f(v - c_0 - c_1 q - p) - (v - c_0 - c_1 q - p)f(v - c_0 - c_1 q - p) \\ &= F(v - c_0 - c_1 q - p)\end{aligned}\tag{33}$$

$$r''(v) = \begin{cases} f(v - c_0 - c_1 q - p) & \text{if } v \neq c_0 + c_1 q + p \\ \text{undefined} & \text{if } v = c_0 + c_1 q + p \end{cases}\tag{34}$$

Since  $r'(v)$  is weakly increasing in  $v$  for  $v \in [0, a]$ ,  $r(v)$  is a convex function. Also, since  $\bar{c} + c_0 > a$ ,  $r''(v)$  is weakly increasing in  $v$  (except for on  $v = c_0 + p$ , where it is undefined).

**Lemma 9.** If  $a = 1$ ,

$$x(p, q) > c_0 + c_1 q + p$$

*Proof.* If  $x \leq c_0 + c_1 q + p$ ,

$$R'(x) = r\left(\frac{x}{2}\right) + \frac{x}{2} r'\left(\frac{x}{2}\right) - r(x) = 0.$$

Since  $K'(x) < 0$ ,  $\pi'(x) > 0$  for all  $x \leq c_0 + c_1q + p$ . Hence,  $x(p) > c_0 + c_1q + p$ .  $\square$

**Proof of Theorem 2:**

Since  $\pi(x; p, q)$  is bounded in  $x \in [0, 1]$ , there exists a  $x(p, q) \in [0, 1]$  that maximizes  $\pi(x; p, q)$ .

Now we need to show that such  $x(p, q)$  is unique. We first show uniqueness when  $a = 1$ . From Lemma 9, such  $x(p) \geq c_0 + c_1q + p > 0$ . Hence, it is sufficient to show that  $\pi(x; p, q)$  is strictly concave in  $x \in [c_0 + c_1q + p, 1]$ . For such  $x$ ,

$$R''(x) = \begin{cases} r'(\frac{x}{2}) + \frac{x}{4}r''(\frac{x}{2}) - r'(x) & \text{if } x \neq 2(c_0 + c_1q + p) \\ \text{undefined} & \text{if } x = 2(c_0 + c_1q + p) \end{cases}$$

If  $x < 2(c_0 + c_1q + p)$ ,

$$\pi''(x) = R''(x) - K''(x) = -F(x - c_0 - c_1q - p) - K''(x) < 0.$$

If  $x > 2(c_0 + c_1q + p)$ ,

$$\begin{aligned} \pi''(x) &= F\left(\frac{x}{2} - c_0 - c_1q - p\right) - F(x - c_0 - c_1q - p) + \frac{x}{4}f\left(\frac{x}{2} - c_0 - c_1q - p\right) - K''(x) \\ &= -\frac{x}{2\bar{c}} + \frac{x}{4\bar{c}} - K''(x) = -\frac{x}{4\bar{c}} - K''(x) < 0 \end{aligned}$$

Since  $\pi''(x) < 0$  almost everywhere in  $[c_0 + c_1q + p, 1]$ ,  $\pi(x)$  is strictly concave. Thus, solution  $x(p, q)$  that maximizes  $\pi(x)$  is unique and is greater than zero.

If  $a \neq 1$ ,  $x(p, q; c, K(x), a) = x\left(\frac{p}{a}, q; \frac{c}{a}, \frac{K(x)}{a}, 1\right)$ , so the optimal  $x(p, q; c, K(x), a)$  exists, and is in  $(0, 1]$ , and is unique.  $\square$

**Lemma 10.** Assume  $a = 1$ . If

$$K'(1) \leq \max\left(-\frac{1}{8\bar{c}}, -\frac{(1 - c_0 - c_1q - p)^2}{2\bar{c}}\right),$$

then  $x(p, q) = 1$ . Otherwise,  $x(p, q) < 1$ .

*Proof.* Since  $x(p, q) > c_0 + c_1q + p$  (Lemma 9), and since  $\pi(x; p, q)$  is strictly concave in  $[c_0 + c_1q + p, 1]$ , if  $\pi'(1; p, q) < 0$ ,  $x(p, q) < 1$ . Otherwise,  $x(p, q) = 1$ .

(i) If  $c_0 + c_1q + p < \frac{1}{2}$ :

$$\pi'(1; p, q) = -\frac{1}{8\bar{c}} - K'(1).$$

Thus, if  $K'(1) > -\frac{1}{8\bar{c}}$ , then  $x(p, q) < 1$ . Otherwise,  $x(p, q) = 1$ .

(ii) If  $c_0 + c_1q + p \geq \frac{1}{2}$ :

$$\pi'(1; p, q) = -\frac{(1 - c_0 - c_1q - p)^2}{2\bar{c}} - K'(1)$$

Thus, if  $K'(1) > -\frac{(1 - c_0 - c_1q - p)^2}{2\bar{c}}$ , then  $x(p, q) < 1$ . Otherwise,  $x(p, q) = 1$ .

Putting them together, we get the Lemma.  $\square$

**Lemma 11.**  $x(p, q)$  is a continuous function in  $p$  and  $q$ .

*Proof.* We first prove the Lemma for  $a = 1$ . We consider different cases depending on the value of  $K'(1)$ .

1. If  $c_0 + c_1q + p < \frac{1}{2}$ :

(a) If  $K'(1) \leq -\frac{1}{8\bar{c}}$ :

From Lemma 10,  $x(p, q) = 1$ , so is continuous in  $p$  and  $q$ .

(b) If  $K'(1) > -\frac{1}{8\bar{c}}$  :

From Lemma 10,  $x(p, q) < 1$ , so  $x(p, q)$  is the solution to  $\pi'(x; p, q) = 0$ . From the Implicit Function Theorem, if  $\pi''(x) \neq 0$ , then  $x(p, q)$  is continuous in  $p$  and  $q$ . If  $x \neq 2(c_0 + c_1q + p)$ , then from the proof of Theorem 2,  $\pi''(x) \neq 0$ . We need to consider what happens if  $x(p, q) = 2(c_0 + c_1q + p)$ .

If there exists  $\hat{p}, \hat{q}$  such that  $\pi'(2(c_0 + c_1\hat{q} + \hat{p}); \hat{p}, \hat{q}) = 0$ , we should show that for a sequence  $\{q_n\} \rightarrow \bar{q}$ , there exists a sequence  $\{x_n\} \rightarrow 2(c_0 + c_1\hat{q} + \hat{p})$  such that  $\pi'(x; \hat{p}, q_n) = 0$ . This means that  $x(\hat{p}, q)$  is continuous at  $\hat{q}$ .

We consider left limit and right limit separately.

Since  $\pi'(2(c_0 + c_1\hat{q} + \hat{p}); \hat{p}, \hat{q}) = 0$ , we get

$$K'(2(c_0 + c_1\hat{q} + \hat{p})) = -\frac{(c_0 + c_1\hat{q} + \hat{p})^2}{2\bar{c}}.$$

For a sequence  $\{q_n\} \rightarrow \hat{q}^-$  :

$$\begin{aligned} \pi'(2(c_0 + c_1\hat{q} + \hat{p}); \hat{p}, q_n) &= r(c_0 + c_1\hat{q} + \hat{p}; \hat{p}, q_n) + \frac{(c_0 + c_1\hat{q} + \hat{p})}{2} r' \left( \frac{c_0 + c_1\hat{q} + \hat{p}}{2}; \hat{p}, q_n \right) \\ &\quad - r(2(c_0 + c_1\hat{q} + \hat{p}); \hat{p}, q_n) - K'(2(c_0 + c_1\hat{q} + \hat{p})) = 0 \end{aligned}$$

Hence, set  $x_n = 2(c_0 + c_1\hat{q} + \hat{p})$ , and the conjecture holds.

For a sequence  $\{q_n\} \rightarrow \hat{q}^+$  :

$$\begin{aligned} \pi'(2(c_0 + c_1\hat{q} + \hat{p}); \hat{p}, q_n) &= -\frac{(c_0 + \hat{p} + 2c_1\hat{q} - c_1q_n)^2}{2\bar{c}} + \frac{(c_0 + c_1\hat{q} + \hat{p})^2}{2\bar{c}} > 0 \\ \pi'(2(c_0 + c_1q_n + \hat{p}); \hat{p}, q_n) &= -\frac{(c_0 + c_1q_n + \hat{p})^2}{2\bar{c}} + \frac{(c_0 + c_1\hat{q} + \hat{p})^2}{2\bar{c}} < 0 \end{aligned}$$

Since  $\pi'(x; p, q)$  is continuous in  $x$ , there exists a  $x_n \in (2(c_0 + c_1\hat{q} + \hat{p}), 2(c_0 + c_1q_n + \hat{p}))$  such that  $\pi'(x_n; \hat{p}, q_n) = 0$ .

As  $\{q_n\} \rightarrow \hat{q}^+$ ,  $\{x_n\} \rightarrow 2(c_0 + c_1\hat{q} + \hat{p})$ . Thus,  $x(\hat{p}, q)$  is continuous at  $\hat{q}$ .

We can show that  $x(p, \hat{q})$  is continuous at  $\hat{p}$  in the exact same way. Thus, overall,  $x(p, q)$  is continuous.

2. If  $c_0 + c_1q + p > \frac{1}{2}$  :

(a) If  $K'(1) < -\frac{(1-c_0-c_1q-p)^2}{2\bar{c}}$  :

From Lemma 10,  $x(p, q) = 1$ , so is continuous in  $p$  and  $q$ .

(b) If  $K'(1) > -\frac{(1-c_0-c_1q-p)^2}{2\bar{c}}$  :

From Lemma 10,  $x(p, q) < 1$ , so  $x(p, q)$  is the solution to  $\pi'(x; p, q) = 0$ . From the Implicit Function Theorem, if  $\frac{\partial \pi'}{\partial x} \neq 0$ , then  $x(p, q)$  is continuous in  $p$  and  $q$ . If  $x \neq 2(c_0 + c_1q + p)$ , then from the proof of Theorem 2,  $\pi''(x) \neq 0$ .

Since  $c_0 + c_1q + p > \frac{1}{2}$ ,  $x(p, q) < 2(c_0 + c_1q + p)$ .

(c) If  $K'(1) = -\frac{(1-c_0-c_1q-p)^2}{2\bar{c}}$  :

From Lemma 10,  $x(p, q) = 1$ , we need to show that if  $\{q_n\} \rightarrow q$ ,  $x(p, q) \rightarrow 1$ .

$$\pi'(1; p, q_n) = -\frac{(1 - c_0 - c_1q_n - p)^2}{2\bar{c}} - K'(1) = -\frac{(1 - c_0 - c_1q_n - p)^2}{2\bar{c}} + \frac{(1 - c_0 - c_1q - p)^2}{2\bar{c}} \rightarrow 0$$

as  $q_n \rightarrow q$ .

3. If  $c_0 + c_1q + p = \frac{1}{2}$  :

Can easily think about left limit (which will fall under the case 1) and right limit (which will fall under case 2) and show that the limits converge correctly and  $x(p, q)$  are continuous.

If  $a \neq 1$ , given

$$x(p, q; c, K(x), a) = x\left(\frac{p}{a}, q; \frac{c}{a}, \frac{K(x)}{a}, 1\right),$$

$x(p, q; c, K(x), a)$  is continuous in  $p$  and  $q$ . □

### A.3 Proof for Symmetric Equilibrium

We first show that it is sufficient to find the symmetric equilibrium for  $a = 1$ .

**Lemma 12.** Denote  $X(c, a, K(x))$  as the symmetric equilibrium  $x$  value under parameters  $c$  and  $a$  and issuance cost  $K(x)$ . Also, denote the TBA market adverse selection measure under the equilibrium as  $S(c, a, K(x)) = s(X(c, a, K(x)); a)$ . Then,

$$X\left(\frac{c}{a}, 1, \frac{K(x)}{a}\right) = X(c, a, K(x)) \quad \text{and} \quad S(c, a, K(x)) = S\left(\frac{c}{a}, 1, \frac{K(x)}{a}\right).$$

*Proof.* This lemma much follow directly from Lemmas 8 and 1 and the definition of adverse selection measure  $s$ . □

#### **Proof of Theorem 3:**

We first show the Theorem for  $a = 1$ . Define  $x^*(y)$  as an originator's optimal response when all other originators choose  $y$ .

$$x^*(y) = x(p(y), q(y))$$

The symmetric equilibrium  $y^*$  satisfies  $y^* = x^*(y^*)$ .

$p(x)$  and  $q(x)$  are continuous functions in  $x$  (Implicit function theorem, using the fact that  $\det(A) \neq 0$  in (31)). From Lemma 11, we know that  $x(p, q)$  is a continuous function. Therefore,  $x^*(y) : [0, 1] \rightarrow [0, 1]$  is a continuous function, and by the Fixed Point Theorem, there exists a fixed point  $y^*$  such that  $x^*(y^*) = y^*$ .

Now we show that such  $y^*$  is unique. Since  $x^*(y)$  is continuous in  $y$ , it is sufficient to show that  $\frac{\partial x^*(y)}{\partial y} < 1$  almost everywhere.

$$\frac{\partial x^*(y)}{\partial y} = \frac{\partial x(p(y), q(y))}{\partial p} \frac{\partial p(y)}{\partial y} + \frac{\partial x(p(y), q(y))}{\partial q} \frac{\partial q(y)}{\partial y}. \quad (35)$$

The above implies that  $y$  affects  $x^*$  only through its effect on  $p(y)$  and  $q(y)$ — i.e. the second-stage equilibrium TBA price and quantity.

1. If  $y < c_0 + c_1 q_0 + p_0$ :

From Lemma 6,  $p(y) = p_0$  and  $q(y) = q_0$ . Therefore,  $\frac{\partial p(y)}{\partial y} = \frac{\partial q(y)}{\partial y} = 0$ , so  $\frac{\partial x^*(y)}{\partial y} = 0$ .

2. If  $y > c_0 + c_1 q_0 + p_0$ :

From Lemma 6,  $y > c_0 + c_1 q(y) + p(y)$ . Thus, using the proof for Proposition 2 (specifically, (31)), we get

$$\begin{pmatrix} \frac{\partial p(y)}{\partial y} \\ \frac{\partial q(y)}{\partial y} \end{pmatrix} = -\frac{1}{\det(A)} \begin{pmatrix} \frac{\partial G(p, q; y)}{\partial q} & -\frac{\partial H(p, q; y)}{\partial q} \\ -\frac{\partial G(p, q; y)}{\partial p} & \frac{\partial H(p, q; y)}{\partial p} \end{pmatrix} \begin{pmatrix} \frac{\partial H(p, q; y)}{\partial y} \\ \frac{\partial G(p, q; y)}{\partial y} \end{pmatrix}.$$

where

$$\begin{aligned} \det(A) &= \frac{\partial H}{\partial p} \frac{\partial G}{\partial q} - \frac{\partial H}{\partial q} \frac{\partial G}{\partial p} \\ &= 1 - \frac{1-y}{\bar{c}}(1+y+c_1) + \frac{(c_0+c_1q+p)(1-y)}{\bar{c}} \left(1 - \frac{c_1(1-y)}{\bar{c}}\right) + \frac{p(1-y)}{\bar{c}} + \frac{c_1(1-y)}{2\bar{c}^2}(1-y^2) \end{aligned}$$

Taking the values of the partial derivatives from the proof of Proposition 2 as well, we get:

$$\frac{\partial p(y)}{\partial y} = -\frac{1}{\det(A)} \frac{y-c_0-c_1q-p}{\bar{c}} \left(-y+p - \frac{c_1+c_1y^2}{2\bar{c}} + \frac{c_1y}{\bar{c}}\right) \quad (36)$$

$$\frac{\partial q(y)}{\partial y} = \frac{1}{\det(A)} \frac{y-c_0-c_1q-p}{\bar{c}} \left(1 - \frac{1-y}{\bar{c}}(1-c_0-c_1q-p)\right) \quad (37)$$

We now calculate  $\frac{\partial x(p,q)}{\partial p}$  and  $\frac{\partial x(p,q)}{\partial q}$ . From Lemma 10, if

$$K'(1) \leq -\frac{1}{8\bar{c}} \text{ or } K'(1) < -\frac{(1-c_0-c_1q-p)^2}{2\bar{c}},$$

$x(p,q) = 1$ , so the partial derivatives are 0. If

$$K'(1) > -\frac{1}{8\bar{c}} \text{ and } K'(1) > -\frac{(1-c_0-c_1q-p)^2}{2\bar{c}},$$

$x(p,q)$  is the solution to  $\pi'(x) = 0$ . From Lemma 9, we have  $x(p,q) > c_0 + c_1q + p$ .

(i) If  $x(p,q) < 2(c_0 + c_1q + p)$ : Using the Implicit Function Theorem, we get

$$\frac{\partial x(p,q)}{\partial p} = \frac{x-c_0-c_1q-p}{x-c_0-c_1q-p + \bar{c}K''(x)} \leq 1 \quad (38)$$

$$\frac{\partial x(p,q)}{\partial q} = \frac{c_1(x-c_0-c_1q-p)}{x-c_0-c_1q-p + \bar{c}K''(x)} \leq c_1 \quad (39)$$

Recall that  $\frac{\partial x(p,q)}{\partial p} > 0$  and  $\frac{\partial x(p,q)}{\partial q} > 0$  from Proposition 2. Putting (36), (37), (38), and (39) into (35), we get

$$\begin{aligned} \frac{\partial x^*(y)}{\partial y} &\leq \frac{1}{\det(A)} \frac{y-c_0-c_1q-p}{\bar{c}} \left[ y-p + \frac{c_1}{\bar{c}}(1-y) \left(\frac{1}{2} - \frac{y}{2}\right) + \frac{c_1(1-y)(y-p)}{\bar{c}} \right. \\ &\quad \left. - c_1 \left(-1 + \frac{1}{\bar{c}}(1-y)(1+y-c_0-c_1q-2p)\right) \right] \\ &= \frac{1}{\det(A)} \frac{y-c_0-c_1q-p}{\bar{c}} \left( y-p + c_1 + \frac{c_1(1-y)}{\bar{c}} \left(-\frac{1}{2} - \frac{y}{2} + c_0 + c_1q + p\right) \right) \end{aligned}$$

$\det(A) > 0$  as shown in the proof of Proposition 13. Thus, to show  $\frac{\partial x^*(y)}{\partial y} < 1$ , it is sufficient to show that

$$\begin{aligned} (y-c_0-c_1q-p) \left( y-p + c_1 + \frac{c_1(1-y)}{\bar{c}} \left(-\frac{1}{2} - \frac{y}{2} + c_0 + c_1q + p\right) \right) \\ < \bar{c} \det(A) = \bar{c} - c_1(1-y) - (1-y)(1+y-c_0-c_1q-2p) + \frac{c_1}{\bar{c}}(1-y)^2 \left(\frac{1}{2} + \frac{1}{2}y - c_0 - c_1q - p\right) \end{aligned} \quad (40)$$

Given that  $y > c_0 + c_1q + p$  and  $(y-c_0-c_1q-p)c_1 < yc_1$ , it is sufficient to show that

$$(y-c_0-c_1q-p)(y-p) < \bar{c} - c_1 - (1-y)(1+y-c_0-c_1q-2p) \quad (41)$$

This is true because

$$\begin{aligned} & \bar{c} - c_1 - (1 - y)(1 + y - c_0 - c_1 q - 2p) - (y - c_0 - c_1 q - p)(y - p) \\ &= \bar{c} - c_1 - 1 + (c_0 + c_1 q + p)(1 - p) + p > 0 \end{aligned}$$

Thus, (40) is true, and thus,  $\frac{\partial x^*(y)}{\partial y} < 1$ .

(ii) If  $x(p, q) \geq 2(c_0 + c_1 q + p)$  :

$$\pi'(x) = r\left(\frac{x}{2}\right) + \frac{x}{2}r'\left(\frac{x}{2}\right) - r(x) = -\frac{x^2}{8\bar{c}} - K'(x).$$

Thus,  $\frac{\partial \pi'(x; p, q)}{\partial p} = \frac{\partial \pi'(x; p, q)}{\partial q} = 0$ , so  $\frac{\partial x(p, q)}{\partial p} = \frac{\partial x(p, q)}{\partial q} = 0$ . Hence,  $\frac{\partial x^*(y)}{\partial y} = 0$ .

Therefore,  $\frac{\partial x^*(y)}{\partial y} < 1$  almost everywhere. Thus, fixed point  $y^*$  is unique.

Next, we show that if

$$K'(1) > -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}, \quad (42)$$

the equilibrium  $y^* < 1$ , and if

$$K'(1) \leq -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}, \quad (43)$$

the equilibrium  $y^* = 1$ .

It is sufficient to show that if everyone else chooses  $y = 1$ ,

a) and if (42) holds, an individual originator chooses  $x^*(1) < 1$ .

b) and if (43) holds, an individual originator chooses  $x^*(1) = 1$ .

If everyone chooses  $y = 1$ , then TBA price is  $p = \frac{1}{2}$ , and TBA quantity is  $q = 1$ . Thus, by Lemma 10, the above conjectures hold.

Lastly, we consider  $a \neq 1$ . Since  $x^*\left(\frac{c}{a}, 1, \frac{K(x)}{a}\right)$  exists and is unique,  $x^*(c, a, K(x))$  exists and is unique. Since

$$K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}} \Leftrightarrow \frac{K'(1)}{a} > -\frac{(\frac{1}{2} - \frac{c_0}{a} - \frac{c_1}{a})^2}{2\bar{c}/a},$$

we get that  $x^*(c, a, K(x)) < 1$  if

$$K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}}$$

and  $x^*(c, a, K(x)) = 1$  otherwise. □

**Lemma 13.** Assume  $a = 1$ . If  $x(p, q) < 2(c_0 + c_1 q + p)$ ,  $x(p, q)$  is strictly increasing in both  $p$  and  $q$ . If  $x(p, q) \geq 2(c_0 + c_1 q + p)$ ,  $x(p, q)$  does not change with  $p$  or  $q$ .

*Proof.* Proof is pretty much contained in the proof for Theorem 3. □

**Proof of Theorem 4:** We first show the theorem for  $a = 1$ . Given that  $\frac{z}{2} \leq p(z)$ ,  $r(\frac{z}{2}; p(z), q(z)) = p(z)$ . From (17),

$$p(z) = \frac{\frac{z^2}{2} + \int_z^1 v(1 - F(v - c_0 - c_1 q(z) - p(z))) dv}{z + \int_z^1 (1 - F(v - c_0 - c_1 q(z) - p(z))) dv}. \quad (44)$$

$$\begin{aligned}
& R(z; p(z)) + (b_0 - m(c_0 + c_1 q)) \\
&= zr \left( \frac{z}{2} \right) + \int_z^1 r(v) dv \\
&= zp(z) + \int_z^1 \left[ p(z) + (v - c_0 - c_1 q(z) - p(z)) F(v - c_0 - c_1 q(z) - p(z)) - \int_0^{v - c_0 - c_1 q(z) - p(z)} \epsilon f(\epsilon) d\epsilon \right] dv \\
&= zp(z) + \int_z^1 p(z) (1 - F(v - c_0 - c_1 q(z) - p(z))) dv + \int_z^1 (v - c_0 - c_1 q(z)) F(v - c_0 - c_1 q(z) - p(z)) dv \\
&\quad - \int_z^1 \int_0^{v - c_0 - c_1 q(z) - p(z)} \epsilon f(\epsilon) d\epsilon dv \\
&= \frac{z^2}{2} + \int_z^1 v (1 - F(v - c_0 - c_1 q(z) - p(z))) dv + \int_z^1 (v - c_0 - c_1 q(z)) F(v - c_0 - c_1 q(z) - p(z)) dv \\
&\quad - \int_z^1 \int_0^{v - c_0 - c_1 q(z) - p(z)} \epsilon f(\epsilon) d\epsilon dv \quad (\because (44)) \\
&= \frac{1}{2} - \int_z^1 (c_0 + c_1 q(z)) F(v - c_0 - c_1 q(z) - p(z)) dv - \int_z^1 \int_0^{v - c_0 - c_1 q(z) - p(z)} \epsilon f(\epsilon) d\epsilon dv \leq \frac{1}{2}
\end{aligned}$$

Thus, we get

$$\begin{aligned}
& R(z; p(z)) \\
&= \frac{1}{2} - \int_z^1 (c_0 + c_1 q(z)) F(v - c_0 - c_1 q(z) - p(z)) dv - \int_z^1 \int_0^{v - c_0 - c_1 q(z) - p(z)} \epsilon f(\epsilon) d\epsilon dv - b_0 + m(c_0 + c_1 q) \\
&\leq \frac{1}{2} - b_0 + m(c_0 + c_1)
\end{aligned}$$

Last inequality holds with equality if and only if  $z = 1$ . Since  $K(z)$  is decreasing in  $z$ ,  $\pi(z) \leq \pi(1)$ , where it holds with equality if and only if  $z = 1$ . Thus, if  $a = 1$ , social planner always chooses  $z = 1$ .

Now we look at cases in which  $a \neq 1$ . Similar to the arguments we have used before, the social planner solution would be the same in the alternative model in which loan dispersion is 1 and the  $c$ ,  $K(x)$ ,  $b_0$  parameters are  $\frac{c}{a}$ ,  $\frac{K(x)}{a}$ , and  $\frac{b_0}{a}$ . Thus, social planner always chooses  $z = 1$ .  $\square$

**Lemma 14.** If  $a = 1$ ,

$$c_0 + c_1 Q(c_0, c_1, \bar{c}) + P(c_0, c_1, \bar{c}) < X(c_0, c_1, \bar{c}) \leq 2P(c_0, c_1, \bar{c}).$$

*Proof.* We know that  $\frac{x}{2} \leq p(x) < 1$  from (3). The lemma follows from this inequality and Lemma 9.  $\square$

**Lemma 15.**  $X(c, a, K(x) = J(x)/\alpha)$  is weakly decreasing in  $\alpha$ , and strictly decreasing if  $\frac{J'(1)}{\alpha} > -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ .

*Proof.* (i) If  $\frac{J'(1)}{\alpha} < -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ :

Then  $X(c, a, K(x) = J(x)/\alpha) = 1$  from Theorem 3. Moreover, there exists a  $\eta > 0$  such that for all  $\hat{\alpha} \in (\alpha - \eta, \alpha + \eta)$ ,  $\frac{J'(1)}{\hat{\alpha}} < -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ , and thus  $X(c, a, K(x) = J(x)/\hat{\alpha}) = 1$ . Therefore,  $\frac{dX(c, a, K(x) = J(x)/\alpha)}{d\alpha} = 0$ .

(ii) If  $\frac{J'(1)}{\alpha} > -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ :

We first show that the best response function,  $x^*(y; c, a, K(x) = J(x)/\alpha)$ , strictly shifts down when  $\alpha$  increases. Given  $y$ , the best response  $x^*(y; c, a, K(x) = J(x)/\alpha)$  is the solution of

$$R'(x; p(y), q(y)) - \frac{J'(x)}{\alpha} = 0$$

. For a given  $y$ ,  $p(y)$  and  $q(y)$  does not change with the issuance cost function. Therefore,  $R'(x; p(y), q(y))$  does not change

with  $\alpha$ . Also, since  $R''(x; p, q) < 0$  almost everywhere,  $'R(x; p(y), q(y))$  is a decreasing function in  $x$ .<sup>28</sup> Since issuance cost function is convex,  $R'(x; p(y), q(y)) - J'(x)/\alpha$  is a decreasing function in  $x$ . Also, since we assume the issuance cost is a decreasing function in  $x$ ,  $R'(x; p(y), q(y)) - J'(x)/\alpha$  shifts down when  $\alpha$  increases, thus the solution  $x^*(y; c, a, K(x) = J(x)/\alpha)$  decreases. Therefore, the best response function strictly shifts down when  $\alpha$  increases. Therefore, the fixed point,  $X(x, a, K(x) = J(x)/\alpha)$ , strictly decreases with  $\alpha$ .

(iii) At  $\frac{J'(1)}{\alpha} = -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ :

The equilibrium  $x$  is 1, and this is the largest possible  $x$ . If  $\alpha$  increases,  $\frac{J'(1)}{\alpha} > -(\frac{1}{2}a - c_0 - c_1)^2/(2\bar{c})$ , so the equilibrium  $x$  will be smaller than 1. If  $\alpha$  decreases, the equilibrium  $x$  will remain 1.

Putting these together completes the proof for the lemma.  $\square$

Lemma 15 shows that decrease in issuance cost (higher  $\alpha$ ) decreases the equilibrium degree of pooling. This is a rather intuitive result. We will use this when we prove the results regarding change in  $a$ .

**Proof of Proposition 4:** We first prove the results regarding  $c$  when  $a = 1$ . If  $K'(1) < -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}}$ , then  $X(c_0, \bar{c}) = 1$ , so

$$\frac{\partial X(c, a)}{\partial c} = 0.$$

We then show that  $X(c_0, c_1, \bar{c})$  is strictly increasing in  $c_0$  if  $K'(1) > -\frac{(\frac{1}{2}a - c_0 - c_1)^2}{2\bar{c}}$ . From Theorem 3, we know that  $X(c_0, c_1, \bar{c}) < 1$ . Since  $X(c_0, c_1, \bar{c})$  is the fixed point  $y^*$  that satisfies  $x^*(y^*; c_0, c_1, \bar{c}) = y^*$ , it is sufficient to show that when we increase  $c_0$ , the best response function  $x^*(y; c_0, c_1, \bar{c})$  is shifted upwards. It is first useful to calculate

$$\begin{aligned} \frac{\partial r(v)}{\partial c_0} &= -F(v - c_0 - c_1 q - p) \\ \frac{\partial r'(v)}{\partial c_0} &= \begin{cases} -f(v - c_0 - c_1 q - p) & \text{if } v \neq c_0 + c_1 q + p \\ \text{undefined} & \text{if } v = c_0 + c_1 q + p \end{cases} \\ \frac{\partial r(v)}{\partial c_1} &= -qF(v - c_0 - c_1 q - p) \\ \frac{\partial r'(v)}{\partial c_1} &= \begin{cases} -qf(v - c_0 - c_1 q - p) & \text{if } v \neq c_0 + c_1 q + p \\ \text{undefined} & \text{if } v = c_0 + c_1 q + p \end{cases} \\ \frac{\partial r(v)}{\partial \bar{c}} &= \begin{cases} 0 & \text{if } v \leq c_0 + c_1 q + p \\ -\frac{(v - c_0 - c_1 q - p)^2}{2\bar{c}^2} & \text{if } v > c_0 + c_1 q + p \end{cases} \\ \frac{\partial r'(v)}{\partial \bar{c}} &= \begin{cases} 0 & \text{if } v \leq c_0 + c_1 q + p \\ -\frac{v - c_0 - c_1 q - p}{\bar{c}^2} & \text{if } v > c_0 + c_1 q + p \end{cases} \end{aligned}$$

We first look at how the best response function changes with  $c_0$ .

$$\frac{\partial x^*(y; c_0)}{\partial c_0} = \frac{dx(p(y; c_0), q(y; c_0); c_0)}{dc_0} = \frac{\partial x(p, q)}{\partial p} \frac{\partial p(y)}{\partial c_0} + \frac{\partial x(p, q)}{\partial q} \frac{\partial q(y)}{\partial c_0} + \frac{\partial x(p, q; c_0)}{\partial c_0}$$

From Lemma 7,  $\frac{\partial p(y)}{\partial c_0} > 0$  and  $\frac{\partial q(y)}{\partial c_0} > 0$ . From Proposition 13, if  $x(p, q) < 2(c_0 + c_1 q + p)$ ,  $\frac{\partial x(p, q)}{\partial p} > 0$  and  $\frac{\partial x(p, q)}{\partial q} > 0$ . If  $x(p, q) \geq 2(c_0 + c_1 q + p)$ , the derivatives are zero.

<sup>28</sup>The proof for  $R''(x; p, q) < 0$  when  $a = 1$  is contained in the proof for Theorem 2. This can be easily shown for  $a \neq 1$  in a similar manner, and we will omit that here.

Since the equilibrium  $x$  is interior, using the fact that  $x(p, q; c_0)$  is the solution to  $\pi'(x; p, q, c_0) = 0$  and the implicit function theorem,

$$\begin{aligned}\frac{\partial x(p, q; c_0)}{\partial c_0} &= -\frac{\frac{\partial \pi'(x(p, q); p, q, c_0)}{\partial c_0}}{\frac{\partial \pi'(x(p, q); p, q, c_0)}{\partial x}} \\ \frac{\partial \pi'(x; p, q, c_0)}{\partial c_0} &= -F\left(\frac{x}{2} - c_0 - c_1 q - p\right) + F(x - c_0 - c_1 q - p) - \frac{x}{2} f\left(\frac{x}{2} - c_0 - c_1 q - p\right) \geq 0 \quad (\text{if } x \neq 2(c_0 + c_1 q + p)) \\ \pi''(x(p, q); p, q, c_0) &= R''(x(p, q)) - K''(x(p, q))\end{aligned}$$

From Lemma 9, we have  $x(p) > c_0 + c_1 q + p$ , so from the proof in Theorem 1,  $\pi''(x(p, q); p, q, c_0) < 0$  as long as  $x(p) \neq 2(c_0 + c_1 q + p)$ . Putting everything together, if  $x < 2(c_0 + c_1 q + p)$ ,  $\frac{\partial x^*(y; c_0)}{\partial c_0} > 0$  almost everywhere. If  $x \geq 2(c_0 + c_1 q + p)$ , we get  $\frac{\partial x^*(y; c_0)}{\partial c_0} = 0$ . Thus, if  $c_0$  increases, the best response function  $x^*(y)$  shifts up weakly, and shifts up strictly in  $x \in [0, 2(c_0 + c_1 q(y) + p(y))]$ . Since  $X(c_0, c_1, \bar{c}) \leq 2P(c_0, c_1, \bar{c})$  (Lemma 14),  $X(c_0, c_1, \bar{c})$  is strictly increasing in  $c_0$ .

Proof for  $c_1$  is exactly the same with minor changes, so we will omit it.

To show that the best response function  $x^*(y)$  shifts up when  $\bar{c}$  increases, we only need to additionally show that  $\frac{\partial \pi'(x; p, q, c_0)}{\partial \bar{c}} \geq 0$  almost everywhere. (Rest of the proof is exactly same as above.)

$$\frac{\partial \pi'(x; p, q, c_0)}{\partial \bar{c}} = \frac{\partial r(\frac{x}{2})}{\partial \bar{c}} - \frac{\partial r(x)}{\partial \bar{c}} + \frac{x}{2} \frac{\partial r'(\frac{x}{2})}{\partial \bar{c}}$$

(i) If  $x(p) \leq 2(c_0 + c_1 q + p)$

$$\frac{\partial \pi'(x; p, q, c_0)}{\partial \bar{c}} = \frac{(x - c_0 - c_1 q - p)^2}{2\bar{c}^2} \geq 0.$$

(ii) If  $x(p) > 2(c_0 + c_1 q + p)$

$$\frac{\partial \pi'(x; p, q, c_0)}{\partial \bar{c}} = \frac{x^2}{8\bar{c}^2} > 0.$$

Hence,  $X(c_0, c_1, \bar{c})$  is weakly increasing in  $\bar{c}$ , and strictly increasing if  $K'(1) > -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}$  and  $x \in [0, 2(c_0 + c_1 q(y) + p(y))]$ .

Since the derivatives are zero if  $K'(1) < -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}$  and positive if  $K'(1) > -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}$ , for  $c_0, c_1$ , and  $\bar{c}$  such that  $K'(1) = -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}$ , the derivatives do not exist.

Now let's show the results regarding change in  $c$  when  $a \neq 1$ . Given Lemma 12 and the above proof for  $a = 1$ , this follows easily.

We now look at the results regarding change in  $a$ . Since

$$X(c_0, c_1, \bar{c}, a, K(x)) = X\left(\frac{c_0}{a}, \frac{c_1}{a}, \frac{\bar{c}}{a}, \frac{K(x)}{a}\right),$$

$$\begin{aligned}\frac{\partial X(c_0, c_1, \bar{c}, a, K(x))}{\partial a} &= -\frac{c_0}{a^2} \frac{\partial X(c_0/a, c_1/a, \bar{c}/a, 1, K(x)/a)}{\partial c_0} - \frac{c_1}{a^2} \frac{\partial X(c_0/a, c_1/a, \bar{c}/a, 1, K(x)/a)}{\partial c_1} - \frac{\bar{c}}{a^2} \frac{\partial X(c_0/a, c_1/a, \bar{c}/a, 1, K(x)/a)}{\partial \bar{c}} \\ &\quad + \left. \frac{dX(c_0/a, c_1/a, \bar{c}/a, 1, K(x)/\alpha)}{d\alpha} \right|_{\alpha=a}\end{aligned}\tag{45}$$

We already know that the first three terms are negative from the earlier results of this Proposition. They are also strictly

negative if

$$\frac{K'(1)}{a} > -\frac{(\frac{1}{2} - c_0/a - c_1/a)^2}{2\bar{c}/a} \Leftrightarrow K'(1) > -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}.$$

We know that the last term is nonpositive from Lemma 15. This completes the proof.  $\square$

**Proof of Proposition 5:** We first prove the results on the change in  $c$  for  $a = 1$ .

$$\frac{dQ(c, a=1)}{dc} = \frac{\partial q(X(c, 1); c, 1)}{\partial x} \frac{\partial X(c, 1)}{\partial c} + \frac{\partial q(X(c, 1); c, 1)}{\partial c} \geq \frac{\partial q(X(c, 1); c, 1)}{\partial c} \geq 0 \quad (46)$$

$$\frac{dS(c, a=1)}{dc} = \frac{\partial s(X(c, 1); c, 1)}{\partial x} \frac{\partial X(c, 1)}{\partial c} + \frac{\partial s(X(c, 1); c, 1)}{\partial c} \leq \frac{\partial s(X(c, 1); c, 1)}{\partial c} \leq 0 \quad (47)$$

(46) and its strict inequality conditions follow from Lemma 14, Proposition 2, Lemma 7, and Proposition 4. (47) and its strict inequality conditions follow from Lemma 14, Propositions 2, 3, and 4. If  $a \neq 1$ , since  $Q(c, a, K(x)) = Q(c/a, 1, K(x)/a)$  and  $S(c, a, K(x)) = S(c/a, 1, K(x)/a)$ :

$$\begin{aligned} \frac{dQ(c, a, K(x))}{dc} &= \frac{1}{a} \frac{dQ(c/a, 1, K(x)/a)}{d(c/a)} \geq \frac{1}{a} \frac{\partial q(X(c/a, 1, K(x)/a); c/a, 1, K(x)/a)}{\partial(c/a)} = \frac{\partial q(X(c, a, K(x)); c, a, K(x))}{\partial(c)} \geq 0 \\ \frac{dS(c, a, K(x))}{dc} &= \frac{1}{a} \frac{dS(c/a, 1, K(x)/a)}{d(c/a)} \leq \frac{1}{a} \frac{\partial s(X(c/a, 1, K(x)/a); c/a, 1, K(x)/a)}{\partial(c/a)} = \frac{\partial s(X(c, a, K(x)); c, a, K(x))}{\partial c} \leq 0 \end{aligned}$$

And the strict inequality conditions follow.

We now look at the results on the change in  $a$ .

$$\frac{dQ(c, a)}{da} = \frac{\partial q(X(c, a); c, a)}{\partial x} \frac{\partial X(c, a)}{\partial a} + \frac{\partial q(X(c, a); c, a)}{\partial a} \leq \frac{\partial q(X(c, a); c, a)}{\partial a} \leq 0 \quad (48)$$

$$\frac{dS(c, a)}{da} = \frac{\partial s(X(c, a); c, a)}{\partial x} \frac{\partial X(c, a)}{\partial a} + \frac{\partial s(X(c, a); c, a)}{\partial a} \geq \frac{\partial s(X(c, a); c, a)}{\partial a} \geq 0 \quad (49)$$

(48) and (49), and their strict inequality conditions follow from Propositions 2, 3, and 4.  $\square$

Lastly, we look at how the equilibrium lender revenue,  $R(c, a)$ , and profit  $\pi(c, a)$  changes with the  $c$  parameters.<sup>29</sup>

**Proposition 6.** *If  $c_0$ ,  $c_1$ , or  $\bar{c}$  increases, the equilibrium lender revenue,  $R(c, a) = R(X(c, a); P(c, a), Q(c, a), c, a)$ , and the equilibrium lender profit,  $\pi$  increases. If  $K'(1) > -\frac{(0.5a - c_0 - c_1)^2}{2\bar{c}}$ ,  $R$  and  $\pi$  increase strictly.*

*Proof.* (For conciseness, we show the proof for  $b_0 = 0$  and  $m = 0$ . It is straightforward to extend to  $b_0 \neq 0$  or  $m \neq 0$ .) We first show the proposition for  $a = 1$ . Since

$$r(v) = p(1 - F(v - c_0 - c_1 q - p)) + (v - c_0 - c_1 q)F(v - c_0 - c_1 q - p) - \int_0^{v - c_0 - c_1 q - p} \epsilon f(\epsilon) d\epsilon,$$

---

<sup>29</sup>A similar result is true for the  $a$  parameter regarding  $R/a$  and  $\pi/a$ .

for  $x \geq c_0 + c_1 q + p$ , for such  $x$ , we can rewrite  $R(x)$  as:

$$\begin{aligned}
R(x) &= xr \left( \frac{x}{2} \right) + \int_x^1 r(v) dv \\
&= xp + p \int_x^1 (1 - F(v - c_0 - c_1 q - p)) dv + \int_x^1 (v - c_0 - c_1 q) F(v - c_0 - c_1 q - p) dv - \int_x^1 \int_0^{v - c_0 - c_1 q - p} \epsilon f(\epsilon) d\epsilon \\
&= \frac{x^2}{2} + \int_x^1 v(1 - F(v - c_0 - c_1 q - p)) dv + \int_x^1 v F(v - c_0 - c_1 q - p) dv \\
&\quad - \int_x^1 \left[ (c_0 + c_1 q) F(v - c_0 - c_1 q - p) + \frac{(v - c_0 - c_1 q - p)^2}{2\bar{c}} \right] dv \quad (\because (17)) \\
&= \frac{1}{2} - \int_x^1 \left[ (c_0 + c_1 q) F(v - c_0 - c_1 q - p) + \frac{(v - c_0 - c_1 q - p)^2}{2\bar{c}} \right] dv
\end{aligned}$$

The first term,  $\frac{1}{2}$ , is the sum of the fundamental value of all loans. The second term is the total trading costs paid by the lender in the SP market.

For ease of exposition, we define  $Z_1(x; c)$  as the  $R$  for a fixed  $x$  and endogenous  $p = p(x; c)$  and  $q = q(x; c)$ . Then we get

$$\frac{dR(X(c); P(c), Q(c), c)}{dc_0} = \frac{dZ_1(X(c); c)}{dc_0} = \frac{\partial Z_1(X(c); c)}{\partial c_0} + \frac{\partial Z_1(X(c); c)}{\partial x} \frac{dX(c)}{dc_0} \quad (50)$$

Since  $X(c) > c_0 + c_1 Q(c) + P(c)$ , we then get

$$\begin{aligned}
\frac{\partial Z_1(X(c); c)}{\partial c_0} &= \int_x^1 \left[ \frac{\partial p(x; c)}{\partial c_0} F(v - c_0 - c_1 q - p) + (c_0 + c_1 q) f(v - c_0 - c_1 - p) \left( 1 + c_1 \frac{\partial q(x; c)}{\partial c_0} + \frac{\partial p(x; c)}{\partial c_0} \right) \right] dv \geq 0 \\
\frac{\partial Z_1(X(c); c)}{\partial x} &= (c_0 + c_1 q) F(x - c_0 - c_1 q - p) + \frac{(x - c_0 - c_1 q - p)^2}{2\bar{c}} \\
&\quad + \int_x^1 \left[ \frac{\partial p(x; c)}{\partial x} F(v - c_0 - c_1 q - p) + (c_0 + c_1 q) f(v - c_0 - c_1 - p) \left( c_1 \frac{\partial q(x; c)}{\partial x} + \frac{\partial p(x; c)}{\partial x} \right) \right] dv > 0
\end{aligned}$$

Since  $\frac{dX(c)}{dc_0} \geq 0$  from Proposition 4, we get  $\frac{dR}{dc_0} \geq 0$ . Also, since  $\frac{dX(c)}{dc_0} > 0$  if  $K'(1) > -\frac{(\frac{1}{2} - c_0 - c_1)^2}{2\bar{c}}$ , we get strict inequality under this condition as well.

The proofs for derivatives on  $c_1$  and  $\bar{c}$  are very similar.

We now consider  $a \neq 1$ . We can easily show that

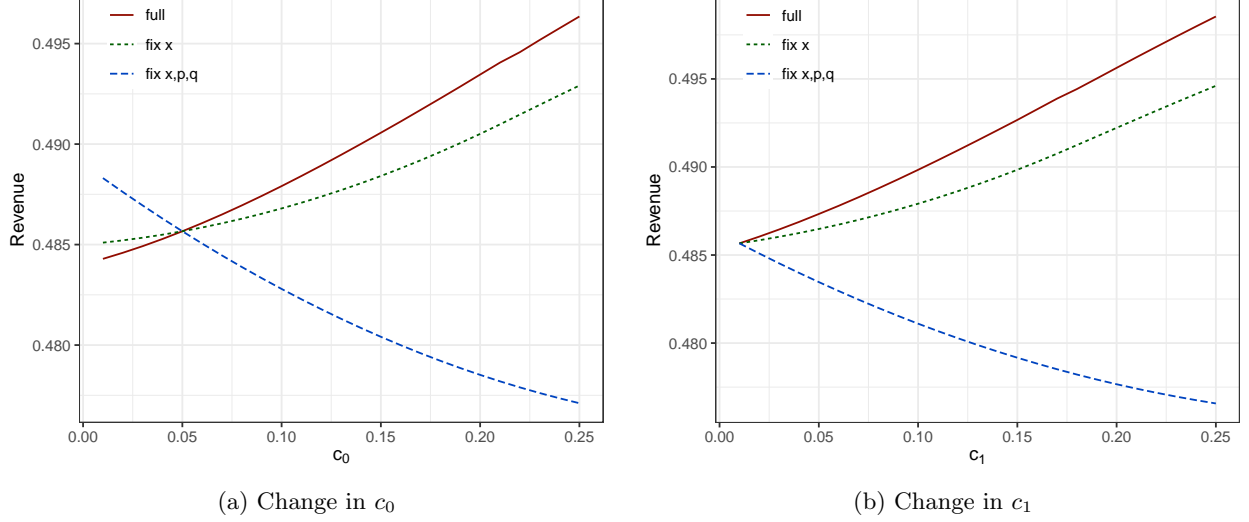
$$R(c, a, K(x)) = aR(c/a, 1, K(x)/a).$$

Since  $R(c/a, 1, K(x)/a)$  is increasing in  $c$ ,  $R(c, a, K(x))$  is increasing in  $c$ . Since the equilibrium  $x$  increases with  $c$ , and  $K(x)$  is a decreasing function in  $x$ ,  $\pi = R - K$  is also increasing in  $c$ .  $\square$

Proposition 6 states that increasing the SP/TBA trading cost difference would increase lenders' revenue and profit from pooling and selling loans in the secondary market. This is a rather surprising result because in the extreme case of  $m = 0$ , holding TBA trading cost fixed, increase SP trading cost would *increase* the lenders' revenues and profits.

To explain the intuition, we provide a simple simulation in Figure A.1. We plot the lender revenue,  $R$ , when  $c_0$  or  $c_1$  changes. In the blue dashed line ("fix  $x$ ,  $p$ ,  $q$ "), we fix the  $x$ ,  $p$  and  $q$  as the equilibrium values at baseline parameters, and plot  $R$  as we vary  $c_0$ . Hence, before, the lender had a choice to sell a pool with value  $v$  either at  $p$  or  $v - \epsilon - c_0$ ; now can sell either at  $p$  or  $v - \epsilon - c_0^{new}$ . Thus, lender revenue  $R$  will decrease in  $c_0$ . In the green dotted line ("fix  $x$ "), we fix  $x$  as the equilibrium value for baseline parameters but let  $p$  and  $q$  vary when  $c_0$  varies. On the one hand, when  $c_0$  increases, trading

Figure A.1: **Lender revenue and  $c$ :** We study how lender revenue ( $R$ ) changes in equilibrium with  $c_0$  and  $c_1$ . In the baseline, we set  $c_0 = 0.05$ ,  $c_1 = 0.01$ ,  $\bar{c} = 1.5$ ,  $K(x) = -0.01x$ , and  $m = 0$ . In the blue dashed line (“fix  $x, p, q$ ”), we fix the  $x, p$ , and  $q$  as the equilibrium values for baseline parameters, and simulate  $R$  as  $c_0$  or  $c_1$  varies. In the green dotted line (“fix  $x$ ”), we fix the  $x$  as the equilibrium  $x$  for the baseline parameters, let  $p$  and  $q$  vary, and simulate  $R$  as we move  $c_0$  or  $c_1$ . The red solid line (“full”) let  $x, p$ , and  $q$  vary. Panel (a) varies  $c_0$ , and Panel (b) varies  $c_1$ .



will shift to TBA market on the margin, and since those pools have values higher than what was originally traded in the TBA market, TBA price  $p$  and  $q$  will increase. This will shift more trading into the TBA market, and will decrease the total trading cost and increase lenders' revenue,  $R$ . On the other hand, SP trading costs will increase due to increase in  $c_0$ , decreasing  $R$ . In our model setup, the first effect dominates, leading  $R$  to increase with  $c_0$ .<sup>30</sup> Lastly, the red solid line plots the change in full equilibrium where  $x, p$ , and  $q$  are allowed to vary with  $c_0$ . Increase in  $p$  and  $q$  leads to increase in  $x$ , which further increases  $p$  and thus  $R$ .

<sup>30</sup>This point is not mentioned in Proposition 6, but is shown in the proof.

## B Supplemental Material for Empirical Tests on Pooling

### B.1 Additional Details about the Empirical Tests

In Section 6, we use three different setups to test the model predictions on MBS pooling. These tests use setups in which there are exogenous differences or changes in loan value dispersion or SP/TBA price differences—comparison across MBS coupons, the 2013 taper tantrum episode, and comparison between Fannie Mae and Freddie Mac. In this section of the Appendix, we show empirically that these setups do result in differences or changes in loan value dispersion or SP/TBA price differences.

#### B.1.1 Loan value dispersion across MBS coupons

We show that within the same agency and maturity, loans in MBS with higher coupons tend to have wider loan value dispersion. Since TBA sellers can deliver any MBS with the specified agency, coupon, and maturity, the applicable loan value distribution would be the distribution of the fundamental value of loans with the specified agency, coupon, and maturity at a given point in time. We measure loan value dispersion as the difference in loan value between small and large loans. Thus, we look at whether this difference is higher for loans in MBS with higher coupons.

Because we do not observe the fundamental values of loans, we make two approximations using the quoted price of MBS from the Fixed Income Data Feed. First is that we use MBS-level data instead of loan-level data. Instead of comparing the values between small and large loans, we compare the values between MBS with small average loan size and MBS with large average loan size. This is relatively innocuous as the value of small loans should be highly correlated with the value of MBS with small average loan size. Second is that we use quoted prices of MBS instead of fundamental values. The distribution of quoted prices is likely different from the distribution of fundamental loan values because the option to trade in the TBA market would impact the former but not the latter. However, wider dispersion in fundamental values would likely lead to wider dispersion in quoted prices. The sample period used to compare distributions for different coupons runs from January to April of 2013 to avoid the taper tantrum episode and capture a period where market interest rates remained relatively steady.

Figure 3 plots the average difference between MBS price and the corresponding TBA price against average loan size for 3%, 3.5%, and 4% coupons. We take the difference between the quoted price for MBS and the corresponding TBA price to remove the time variation in MBS prices due to changes in prevailing mortgage rates or other market factors. This figure shows that the price differences between MBS with small average

loan size and MBS with large average loan size are larger for higher coupons. For instance, in Fannie Mae 30-year MBS, prices for 4% MBS with average loan size around \$100,000 are approximately 2 percentage points higher than those with average loan size around \$300,000, whereas the corresponding value for 3.5% MBS is about 1 percentage point. Therefore, loan value dispersion is higher for loans in higher-coupon MBS.

### B.1.2 Loan value dispersion before and after taper tantrum

We first show that, for a given coupon rate, loan value distribution shrinks after TT. An increase in prevailing mortgage rate would decrease refinance incentives, shrinking the prepayment differences between small and large loans for a given MBS coupon. Thus, loan values would become less dispersed. As in Section B.1.1, we proxy for loan value dispersion with the difference in quoted prices between MBS with small average loan size and large average loan size. We run the following regression using the Fixed Income Data Feed data. We use all 30-year TBA-eligible MBS that were issued in the last 30 days, and the sample runs from January to December of 2013.

$$y_{m,t} = \sum_j \beta_{j,\tau} \mathbb{1}[avgbal_m \in G_j] + \xi_{agency} + \xi_{j,coupon} + \xi_{j,\tau} + \epsilon_{m,t} \quad (51)$$

The dependent variable is negative of the daily difference between MBS price and its corresponding TBA price. More precisely,  $y_{m,t} = -v_{m,t} + TBA_{m,t}$ , where  $v_{m,t}$  is the price of MBS  $m$  on day  $t$ , and  $TBA_{m,t}$  is the price of the corresponding TBA contract.<sup>31</sup>  $t$  stands for day, and  $\tau$  stands for month.  $\mathbb{1}[avgbal_m \in G_j]$  is a dummy variable that equals 1 if  $avgbal$  belongs to group  $j$  ( $G_j$ ). There are five groups:  $[0, 100K]$ ,  $(100K, 200K]$ ,  $(200K, 300K]$ ,  $(300K, 400K]$ ,  $[400K, Inf]$ .

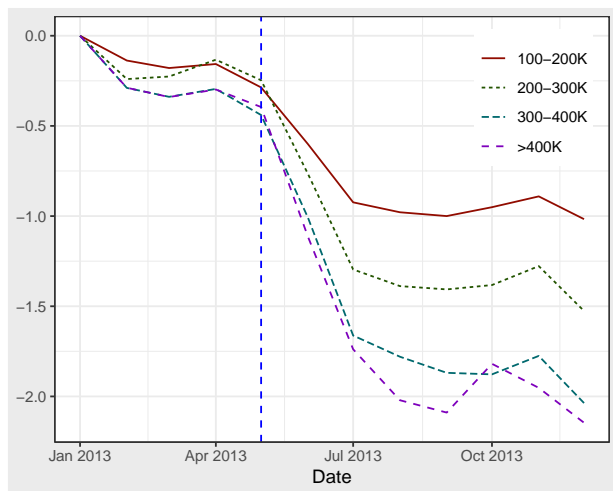
In estimating  $\beta_{j,\tau}$ , we use January 2013 and  $avgbal$  of  $[0, 100K]$  as the omitted categories. Hence, for a given month,  $\beta_{j,\tau}$  measures the average difference in prices between MBS in  $G_1$  and MBS in  $G_j$ , which can be thought of as a measure of loan value dispersion. Then by comparing estimates of  $\beta_{j,\tau}$  across  $\tau$  for a given  $j$ , we can see how loan value dispersion changed over time. A negative value of  $\beta_{j,\tau}$  means that the dispersion decreased compared to January. We also control for agency fixed effects,  $avgbal$  group  $\times$  coupon fixed effects, and  $avgbal$  group  $\times$  month fixed effects.

Figure A.2 plots the results. Panel (a) plots the estimated betas for each  $avgbal$  groups. Betas drop sharply after May and are negative, which means that the loan value distribution narrows significantly around the taper tantrum event. Panel (b) shows the beta estimates for  $[300K, 400K]$   $avgbal$  group and

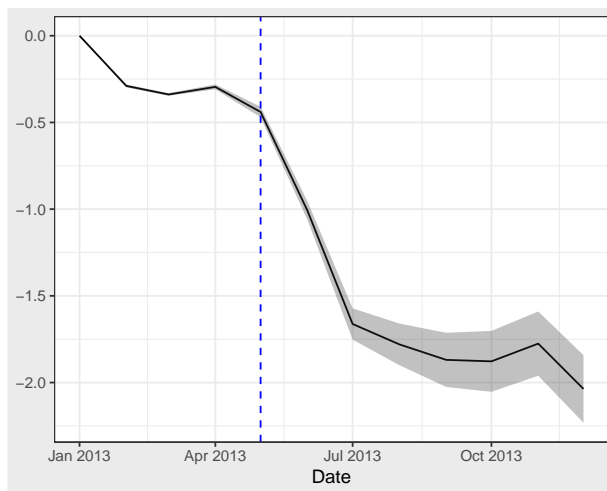
<sup>31</sup>Because we are interested in the value of smallest  $avgbal$  group MBS minus the value of the  $j$ -th group, and because the smallest  $avgbal$  group is the omitted category, we define  $y_{m,t}$  in this way.

Figure A.2: **Loan value dispersion around Taper Tantrum**

The following graphs plot the estimated  $\beta$  values from regression (51). Panel (a) plots  $\beta$  separately for each size bin. Panel (b) plots the  $\beta$  estimates and their 95% confidence intervals for the [\$300,000, \$400,000] average loan size group. Source: ICE Data Pricing & Reference Data, LLC.



(a) Regression results



(b) Beta estimates for [300K, 400K) group

their 90th percentile confidence intervals. The change is statistically significant.

### B.1.3 Fannie Mae v Freddie Mac

Figure 5 plots the monthly turnover for Fannie Mae and Freddie Mac 30 year newly-issued MBS calculated using the regulatory TRACE data. For the denominator in both TBA and SP turnover calculations, we use the monthly issuance amount of TBA-eligible MBS.<sup>32</sup> For SP turnover numerator, we use trades of TBA-eligible MBS. Panel (a) shows that the turnover for Fannie Mae TBAs is significantly higher than that of Freddie Mac TBAs, implying higher liquidity for Fannie Mae TBAs. On the other hand, as shown in Panel (b), SP turnover is similar for Fannie Mae and Freddie Mac MBS. Since trading volume and liquidity are highly correlated, these results imply that the difference between SP and TBA trading cost is higher for Fannie MBS than for Freddie MBS.

## B.2 Degree of Separation with Respect to LTV

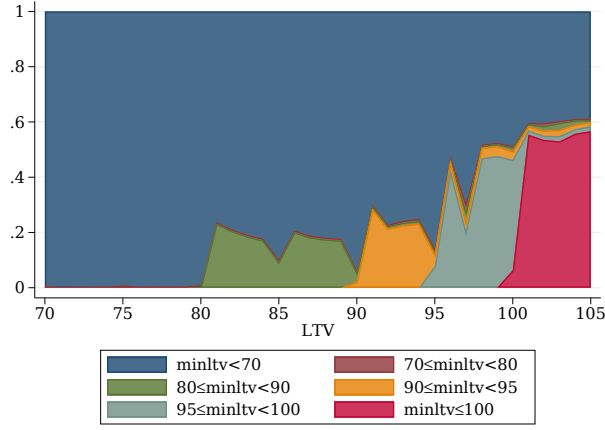
Although loan size is the most important determinant of expected prepayments and loan values, other loan characteristics matter as well. In this section, we focus on the LTV at origination. High-LTV loans—loans with LTVs greater than 80—tend to prepay slower because borrowers of such loans typically take a longer time to build equity in their houses and have low enough LTVs to be able refinance to lower rates. To characterize lenders’ pooling practices with regards to LTVs, we define  $minltv_m$  as the minimum LTV of loans in MBS  $m$ . Figure A.3 shows that lenders appear to pool loans with similar values for LTVs greater than 80. These high-LTV MBS—those with  $minltv_m$  of at least 80—account for 5% of MBS issuance during our sample period.<sup>33</sup>

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<sup>32</sup>TBA volume tends to correlate highly with recent issuance amount.

<sup>33</sup>For a comparison, low-balance MBS—those with  $minbal_m$  up to \$175,000—account for 20%.

Figure A.3: **Share in MBS Minimum LTV Bin:** This figure displays the cumulative share of loans in MBS with different  $minltv_m$ . Data source: eMBS.



We repeat the three tests in Section 6 using LTV as a proxy for loan values instead of loan size. Specifically, we estimate the following equation that is similar to Equation (6).

$$1[minltv_{m(i)} > 80] = X_i(\beta) + z_i\gamma + \xi_i + \epsilon_i \quad (52)$$

The dependent variable is a dummy variable that is equal to one if the minimum LTV of loans included in MBS  $m(i)$  is strictly greater than 80. All other variables are defined in the same way as in Equation (6). We estimate Equation (52) using loans with LTVs greater than 80 because the dependent variable is always zero for other loans. The estimates for 15- and 30-year MBS are reported in Tables A.1 and A.2, respectively. We find that the results of the three tests are consistent with the model predictions.

Table A.1: **Testing for Degree of Separation with Respect to LTV at Origination (15-year MBS)** This table shows coefficient estimates for the regression in Equation (52). We estimate the regression separately only for loan included in 15-year MBS. Column (1) compares the degree of separation across MBS with coupons of 2.5% and 3%. Column (2) compares the degree of separation between periods before and after the TT event. Column (3) compares the degree of separation between Fannie Mae and Freddie Mac MBS. Standard errors are clustered at the same level as the fixed effect. Data source: eMBS.

	(1)	(2)	(3)
Coupons:			
1[Coupon=3%]	0.129*** (0.004)		
Taper Tantrum:			
1[OriginationMonth $\geq$ 2013:m5]		-0.057*** (0.006)	
Fannie v Freddie:			
1[Agency=Freddie]			0.049*** (0.008)
Agency x Time x State x Lender x Loan Size Bin	Y		
Coupon x Agency x State x Lender x Loan Size Bin		Y	
Coupon x Time x State x Lender x Loan Size Bin			Y
Other Controls	Y	Y	Y
N. Obs.	119,018	40,492	107,440
Adj. $R^2$	0.63	0.59	0.68

Table A.2: **Testing for Degree of Separation with Respect to LTV at Origination (30-year MBS)** This table shows coefficient estimates for the regression in Equation (52). We estimate the regression separately only for loan included in 30-year MBS. Column (1) compares the degree of separation across MBS with coupons of 3%, 3.5%, and 4%. Column (2) compares the degree of separation between periods before and after the TT event. Column (3) compares the degree of separation between Fannie Mae and Freddie Mac MBS. Standard errors are clustered at the same level as the fixed effect. Data source: eMBS.

	(1)	(2)	(3)
Coupons:			
1[Coupon=3.5%]	0.196*** (0.002)		
1[Coupon=4%]	0.316*** (0.002)		
Taper Tantrum:			
1[OriginationMonth $\geq$ 2013:m5]		-0.113*** (0.002)	
Fannie v Freddie:			
1[Agency=Freddie]			0.073*** (0.003)
Agency x Time x State x Lender x Loan Size Bin	Y		
Coupon x Agency x State x Lender x Loan Size Bin		Y	
Coupon x Time x State x Lender x Loan Size Bin			Y
Other Controls	Y	Y	Y
N. Obs.	1,578,349	338,226	1,071,635
Adj. $R^2$	0.61	0.59	0.65

## C Additional Figures

Figure A.4: **Loan Size and Ex-post Prepayments:** These figures display the relationship between ex-post prepayments and loan size. We use a sample of 30-year fixed-rate purchase loans. The loan amount in the x-axis is measured in thousands of dollars. Ex-post prepayments in the y-axis are measured in terms of whether a loan was paid off completely by loan age of 24 months (Panel (a)) and 48 months (Panel (b)) since origination. To control for potentially different prepayment behaviors depending on when a loan is originated and other loan characteristics, we consider residual prepayments, which are calculated by removing variation accounted for by lender fixed effects and origination year-month, mortgage rate bin fixed effects, as well as state fixed effects. We use 25 bps for mortgage rate bins. Data Sources: Fannie Mae and Freddie Mac Single-Family Loan-Level Data.

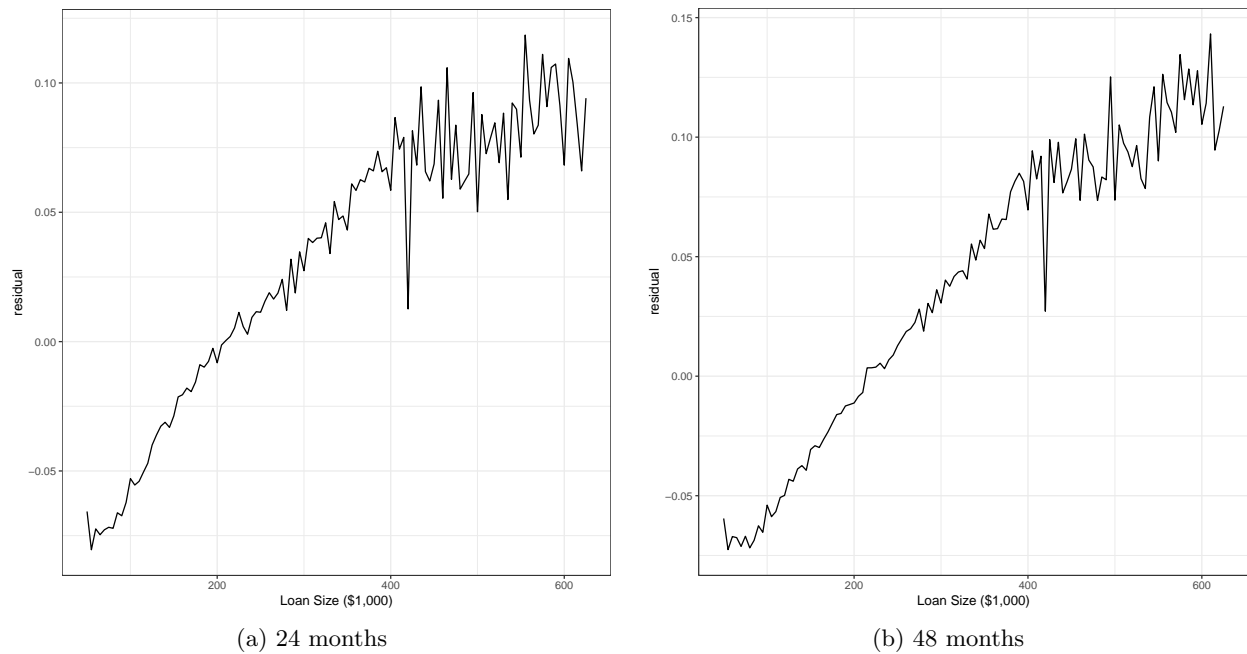


Figure A.5: **Maximum original loan size distribution of MBS traded in TBA and SP markets**  
 We plot the distribution of *maxbal* for TBA and SP markets. We back out TBA settlement from SOMA holdings data. SP trade data is from the regulatory TRACE data. The vertical line is at \$175,000.

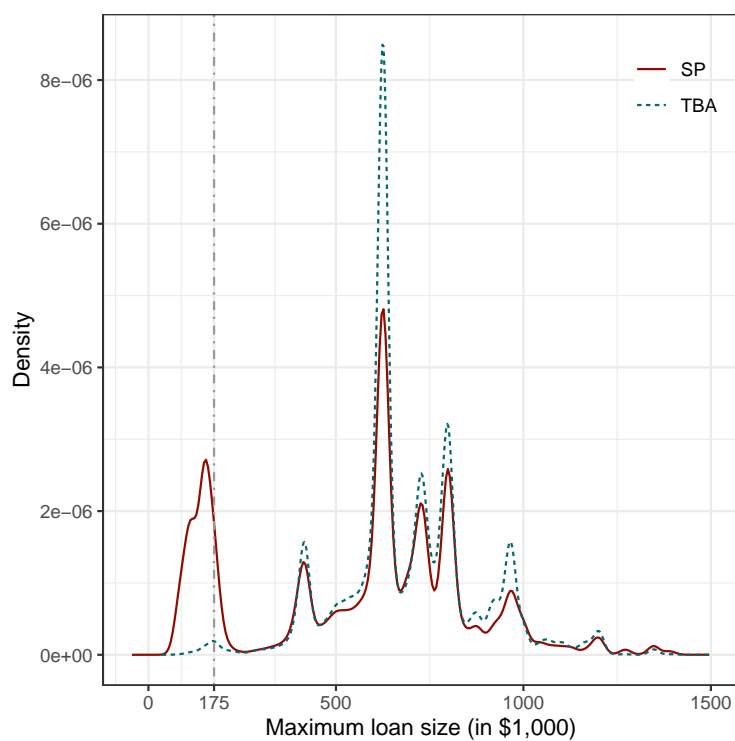
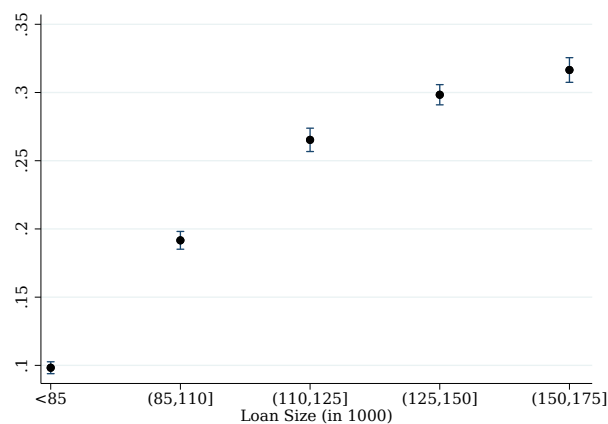
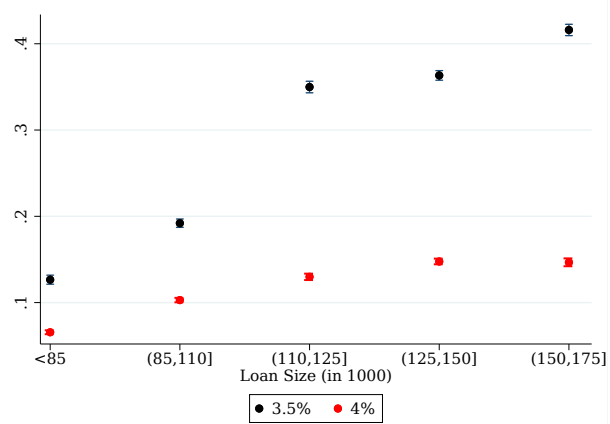


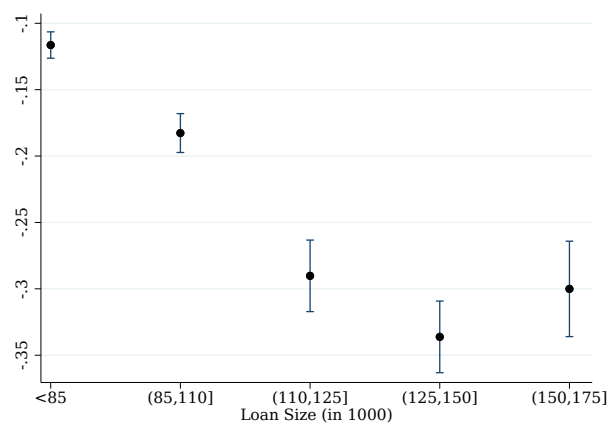
Figure A.6: **Estimates by Loan Size Bin:** This figure displays the coefficient estimates and their 95% confidence intervals for regression (6) that allows for different  $\beta$  for each loan size bin. The estimates are plotted separately for different coupons (Panels (a) and (b)), before/after TT (Panels (c) and (d)), and for Fannie and Freddie (Panels (e) and (f)). Data source: eMBS.



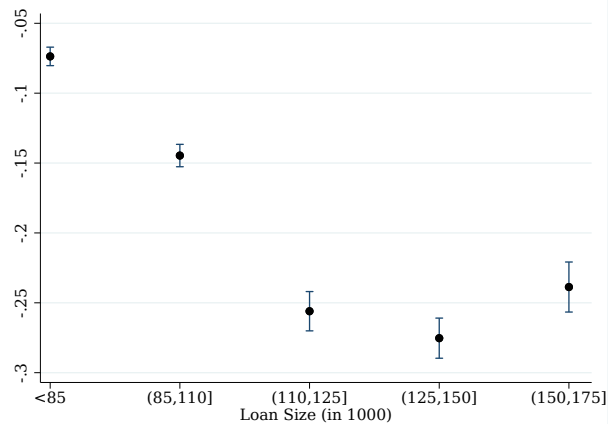
(a) Across Coupons (15 Years)



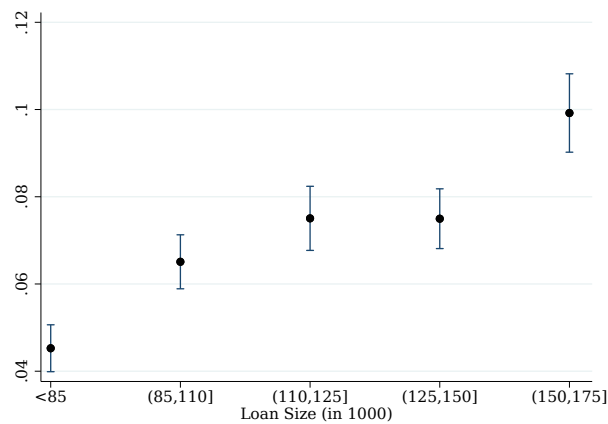
(b) Across Coupons (30 Years)



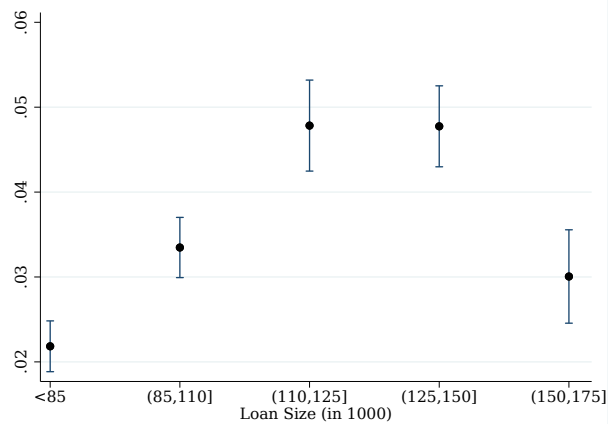
(c) Taper Tantrum (15 Years)



(d) Taper Tantrum (30 Years)



(e) Fannie v Freddie (15 Years)



(f) Fannie v Freddie (30 Years)

## D Additional Tables

Table A.3: **Testing for Degree of Separation (15-year MBS)** This table shows coefficient estimates for the regression in Equation (6). We estimate the regression separately only for loan included in 15-year MBS. Column (1) compares the degree of separation across MBS with coupons of 2.5% and 3%. Column (2) compares the degree of separation between periods before and after the TT event. Column (3) compares the degree of separation between Fannie Mae and Freddie Mac MBS. Standard errors are clustered at the same level as the fixed effect. The stars indicate: \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ . Data source: eMBS.

	(1)	(2)	(3)
Coupons:			
1[Coupon=3%]	0.205*** (0.002)		
Taper Tantrum:			
1[OriginationMonth $\geq$ 2013:m5]		-0.218*** (0.005)	
Fannie v Freddie:			
1[Agency=Freddie]			0.067*** (0.003)
Agency x Time x State x Lender x Loan Size Bin	Y		
Coupon x Agency x State x Lender x Loan Size Bin		Y	
Coupon x Time x State x Lender x Loan Size Bin			Y
Other Controls	Y	Y	Y
N. Obs.	893,240	227,643	701,370
Adj. $R^2$	0.70	0.53	0.71